Numerical Methods 1
Exam formula sheet

Floating point numbers

Binary floating point numbers have the form:

$$s1.m \times 2^{e-b}$$

Where:

- s is the sign (0 for +, 1 for -).
- $1.m$ is the mantissa.
- e is the exponent.
- b is the bias.

s,m,e, and b are written in binary.

The bit pattern of a floating point number with 3 exponent bits and 4 mantissa bits is:

s	e	e	e	1	m	m	m	m
$\pm$	2^2	2^1	2^0	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}

where the leading 1 in the mantissa is not actually stored.

Taylor series

$$f(x+h) = \sum_{k=0}^{n} \frac{h^k}{k!} f^{(k)}(x) + \mathcal{O}(h^{n+1})$$

Matrix operations

The following are true for any matrices or vectors A,B,C for which the corresponding dimensions match and for any scalar α:

- $\mathrm{A} \cdot (\mathrm{B} \cdot \mathrm{C}) = (\mathrm{A} \cdot \mathrm{B}) \cdot \mathrm{C}$
- $\mathrm{A} \cdot (\mathrm{B} + \mathrm{C}) = \mathrm{A} \cdot \mathrm{B} + \mathrm{A} \cdot \mathrm{C}$
- $(\mathrm{A} + \mathrm{B}) \cdot \mathrm{C} = \mathrm{A} \cdot \mathrm{C} + \mathrm{A} \cdot \mathrm{C}$
- $\mathrm{A} + \mathrm{B} = \mathrm{B} + \mathrm{A}$
- $\alpha\mathrm{A} \cdot \mathrm{B} = \mathrm{A} \cdot \alpha\mathrm{B}$

Linear independence

For a square matrix A, the following statements are equivalent:

1. The columns of A are linearly independent.
2. The span of the columns of A is $\mathbb{R}^n$.
3. A is invertible. i.e. there exists a matrix A^{-1} such that $AA^{-1} = I$
4. $A\mathbf{x} = \mathbf{b}$ has a unique solution for every $\mathbf{b} \in \mathbb{R}^n$.
5. The unique solution to $A\mathbf{x} = \mathbf{0}$ is the zero vector, $\mathbf{0}$.
6. $\det(A) \neq 0$.
7. 0 is not an eigenvalue of A.
8. A has full rank.