# A User's Guide to gringo, clasp, clingo, and iclingo \*

(version 3.x)

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## - Preliminary Draft -

#### Abstract

This document provides an introduction to the Answer Set Programming (ASP) tools gringo, clasp, clingo, and iclingo, developed at the University of Potsdam. The first tool, gringo, is a *grounder* capable of translating logic programs provided by users into equivalent propositional logic programs. The answer sets of such programs can be computed by clasp, which is a *solver*. The third tool, clingo, integrates the functionalities of gringo and clasp, thus, acting as a *monolithic* solver for user programs. Finally, iclingo extends clingo by an *incremental* mode that incorporates both grounding and solving. For one, this document aims at enabling ASP novices to make use of the aforementioned tools. For another, it provides a reference of their features that ASP adepts might be tempted to exploit.

Note that this document contains a lot of examples. For convienience no examples have to be typed in by hand instead they can directly be safed to disc by clicking them.

\*Tools gringo, clasp, clingo, and iclingo are available at [46].

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# **1** Introduction

The "Potsdam Answer Set Solving Collection" (Potassco) [46] by now gathers a variety of tools for Answer Set Programming. Among them, we find grounder gringo, solver clasp, and combinations thereof within integrated systems clingo and iclingo. All these tools are written in C++ and published under GNU General Public License(s) [30]. Source packages as well as precompiled binaries for Linux and Windows are available at [46]. For building one of the tools from sources, please download the most recent source package and consult the included README or INSTALL text file, respectively. Please make sure that the platform to build on has the required software installed. If you nonetheless encounter problems in the building process, please use the potassco mailing list potassco-users@lists.sourceforge.net or consult the supporting pages at potassco.sourceforge.net.

After downloading (and possibly building) a tool, one can check whether everything works fine by invoking the tool with flag --version (to get version information) or with flag --help (to see the available command line options). For instance, assuming that a binary called gringo is in the path (similarly, with the other tools), the following command line calls should be responded by gringo:

```
gringo --version
gringo --help
```

If grounder gringo, solver clasp, as well as integrated systems clingo and iclingo are all available, one usually provides the file names of input text files to either gringo, clingo, or iclingo, while the output of gringo is typically piped into clasp. Thus, the standard invocation schemes are as follows:

```
gringo [ options | files ] | clasp [ options | number ]
clingo [ options | files | number ]
iclingo [ options | files | number ]
```

Note that a numerical argument provided to either clasp, clingo, or iclingo determines the maximum number of answer sets to be computed, where 0 stands for "compute all answer sets." By default, only one answer set is computed (if it exists).

This guide introduces the fundamentals of using gringo, clasp, clingo, and iclingo. In particular, it tries to enable the reader to benefit from them by significantly reducing the "time to solution" on difficult problems. The outline is as follows. In Section 2, an introductory example is given that serves both as guideline on how to model problems using logic programs and also as an example on how compact and concise the modeling language of gringo is. The probably most important part for a user, Section 3, is dedicated to the input languages of our tools, where the joint input language of gringo and clingo claims the main share (later on, it is extended by iclingo). For illustrating the application of our tools, three well-known example problems are solved in Section 4. Practical aspects are also in the focus of Section 5 and 6, where we elaborate and give some hints on the available command line options as well as input-related errors and warnings that may be reported. During the guide we forgo most of the theoretical background in favor of small intuitive examples and informal descriptions.

For readers familiar with lparse [53] (a grounder that constitutes the traditional front end of solver smodels [51]), Appendix A lists the most prominent differences to our tools. Otherwise, gringo, clingo, and iclingo should accept most inputs recognized by lparse, while the input of solver clasp can also be generated by

lparse instead of gringo. Throughout this guide, we provide quite a number of examples. Many of them can actually be run, and instructions on how to accomplish this (or sometimes meta-remarks) are provided in margin boxes, where an occurrence of "\" usually means that a text line broken for space reasons is actually continuous. After all these preliminaries, it is time to start our guided tour through Potassco [46]. We hope that you will find it enjoyable and helpful!



Figure 1: Towers of Hanoi Initial Situation

# 2 Quickstart

In this section we demonstrate the expressive power and the simple yet powerful modeling language of gringo by looking at the simple Towers of Hanoi puzzle. It consists of three pegs and a set of discs of different sizes, which can be put onto the pegs. The goal is to move all pegs from the leftmost peg to the rightmost peg, where at each time only the topmost disc can be moved on top of another peg. Additionally, a disc may not be put on top of a smaller disc. We ignore that there is an efficient algorithm to solve this problem and just specify how a solution, in terms of a sequence of moves, has to look.

In ASP it is custom to provide a *uniform* problem definition [39, 42, 50]. Following this methodology, we separate the encoding from an instance of the following problem: given an initial placement of the discs, a goal situation, and a number n, decide whether there is a sequence of moves of length n that satisfies the conditions given above. We will see that this decision problem can be elegantly specified by reducing it to a declarative problem solving paradigm like ASP, where efficient off-the-shelf tools like gringo and clasp are ready to solve the problem reasonably well. Such a reduction is now exemplified.

## 2.1 Problem Instance

We consider a Towers of Hanoi instance specified via facts over predicates peg/1 and disk/1 that correspond to the pegs and disks in the puzzle. Discs are enumerated by consecutive integers beginning with one, where a disc with a lower number is considered to be bigger than a disc with a higher number. The pegs can have arbitrary names. Furthermore, the predicates init\_on/2 and goal\_on/2 describe the initial and goal situation, respectively. Their first argument is the number of a disc and the second argument is the peg on which the disc is located in the initial or goal situation. Finally, the predicate moves/1 specifies the number of moves within which the goal situation has to be reached. Note that the original puzzle had exactly three pegs and a fixed initial and goal situation. With ASP we can easily change this requirement and the encoding represented in the following works with an arbitrary number of pegs and any initial or goal situation. Figure 1 depicts a possible instance (the dashed discs mark the goal situation) corresponding to the ASP program given below.

```
1 peg(a;b;c).
```

```
2 disk(1..4).
```

```
3 init_on(1..4,a).
```

```
4 goal_on(1..4,c).
```

```
5 moves(15).
```

The ";" in the first line is some syntactic sugar (Section 3.1.9) that expands the statement into three facts peg(a), peg(b), and peg(c) representing the three pegs. Again, in the second line some syntactic sugar is used to create the facts disc(1), disc(2), disc(3), and disc(4). Here the term 1..4, an intervall (Section 3.1.7), is successively replaced by 1, 2, 3, and 4. The initial and goal situation is specified in line three and four again using intervall. Finally, in the last line the number of moves to solve the problem is given.

### 2.2 Problem Encoding

We now proceed by encoding the Towers of Hanoi puzzle via non-ground rules (Section 3.1.1), i.e, rules with variables that are independent of particular instances. Typically, an encoding consists of a *Generate*, a *Define*, and a *Test* part [36]. We follow this paradigm and mark respective parts via comment lines beginning with % in the encoding below. The variables D, P, T, and M are used to refer to disks, pegs, the T-th move in the sequence of moves, and the length of the sequence, respectively.

```
1
   % Generate
2
   1 { move(D,P,T) : disk(D) : peg(P) } 1 :- moves(M), T = 1..M.
  % Define
3
4 move(D,T)
               :- move(D,_,T).
5 on (D, P, 0)
               :- init_on(D,P).
6 on (D, P, T)
             :- move(D,P,T).
7 on (D, P, T+1) :- on (D, P, T), not move (D, T+1), not moves (T).
8 blocked(D-1,P,T+1) :- on(D,P,T), disk(D), not moves(T).
9 blocked(D-1,P,T) :- blocked(D,P,T), disk(D).
10 % Test
11
  :- move(D,P,T), blocked(D-1,P,T).
12 :- move(D,T), on(D,P,T-1), blocked(D,P,T).
13 :- goal_on(D,P), not on(D,P,M), moves(M).
14 :- not 1 { on(D,P,T) : peg(P) } 1, disk(D), moves(M), T = 1..M.
15 #hide.
16 #show move/3.
```

The Generate part consists of just one rule in Line 2. At each time point T at which a move is executed, we "guess" exactly one move that puts an arbitrary disk to some arbitrary peg. The head of this rule is a so called cardinality constraint (Section 3.1.10) that consists of a set that is expanded using the predicates behind the colons (Section 3.1.8) and a lower and an upper bounds. The constraint is true if and only if the number of true literals within the set is between the upper and lower bound. Furthermore, the constraint is used in the head of a rule, that is, it is not only a test but can derive ("guess") new atoms, which in this case correspond to possible moves of discs. Note that at this point we have not constrained the moves. Up to now, any disc could be moved to any peg at each time point with out considering any problem constraints.

Next follows the Define part, here we give rules that define new auxiliary predicates, which as such do not touch the satisfiability of the problem but are used in the Test part later on. The rule in Line 4 projects out the target peg of a move, i.e., the predicate move/2 can be used if we only need the disc affected by a move but not its target location. We use the predicate on/3 to capture the state of the Hanoi puzzle at each time point. Its first two argument give the location of a disc at the time point given by the third argument. The next rule in Line 5 infers the location of each disc in the initial state (time point 0). Then we model the state transition using the rules in Line 6 and 7. The first rule is quite straightforward and states that the moved disc changes its location. Note the usage of not moves (T) here. This literal prevents deriving an infinite number of rules, which would be all useless because the state no longer changes after the last move. The second rule makes sure that all discs that are not moved stay where they are. Finally, we define the auxiliary predicate blocked/3, which marks positions w.r.t. pegs that cannot be moved from. First in Line 8, the position below a disc on some peg is blocked. Second in Line 9, the position directly below a blocked position is blocked. Note that we mark position zero to be blocked, too. This is convenient later on to assert some redundant moves.

Finally, there is the Test part building upon both Generate and Define part to rule out wrong moves that do not agree with the problem description. It consists solely of integrity constraints, which fail whenever all their literals are true. The first integrity constraint in Line 11 asserts that a disc that is blocked, i.e., with some disc on top, cannot be moved. Note the usage of D-1 here, this way a disc cannot be put back to the same location again. The integrity constraint in Line 12 asserts that a disc can only be placed on top of a bigger disc. Line 13 asserts the goal situation. To make the encoding more efficient, we add a redundant constraint in Line 14, which asserts that each disc at all time points is located on exactly one peg. Although, this constraint is implied by the constraints above, adding this additional domain knowledge greatly improves the speed with which the problem can be solved. Finally, the last two statements control which predicates are printed, when a satisfying model for the instance is found. Here we first hide all predicates (Line 15) and then explicitly show only the move/3 predicate (Line 16).

## 2.3 Problem Solution

Now we are ready to solve the encoded puzzle. To find an answer set, invoke one of the following commands (clingo, or gringo and clasp have to be installed somewhere under the systems path for the commands below to work):

```
clingo inst_toh.lp enc_toh.lp
gringo inst_toh.lp enc_toh.lp | clasp
```

Note that (depending on your viewer) you can right or double-click on file names marked with a red font to safe the associated file to disc. This is possible with all examples given in this document.

The output of the solver (clingo in this case) looks something like that:

```
Answer: 1
move(4, b, 1)
             move(3,c,2)
                          move(4,c,3)
                                        move(2, b, 4)
                                                      \
move(4,a,5) move(3,b,6)
                          move(4,b,7)
                                        move(1,c,8)
                                                      \
move(4,c,9)
             move(3,a,10) move(4,a,11) move(2,c,12) \
move(4,b,13) move(3,c,14) move(4,c,15)
SATISFIABLE
Models
            : 1+
```

```
Time : 0.010

Prepare : 0.000

Prepro. : 0.010

Solving : 0.000
```

The first line indicates that an answer set follows in the line below (the  $\ marks$  a line wrap). Then the status follows, this might be either SATISFIABLE, UNSATISFIABLE, or UNKNOW if the computation is interrupted. The 1+ right of Models: indicates that one answer set has been found and the + that the whole search space has not yet been explored, so there might be further answer sets. Following that, there are some time measurements: Beginning with total computation time, which is split into preparation time (grounding), preprocessing time (clasp has an internal preprocessor that tries to simplify the program), and solving time (the time needed to find the answer excluding preparation and preprocessing time). More information about options and output can be found in Section 5.

# **3** Input Languages

This section provides an overview of the input languages of grounder gringo, combined grounder and solver clingo, incremental grounder and solver iclingo, and of solver clasp. The joint input language of gringo and clingo is detailed in Section 3.1. It is extended by iclingo with a few directives described in Section 3.2. Finally, Section 3.3 is dedicated to the inputs handled by clasp.

## 3.1 Input Language of gringo and clingo

The tool gringo [26] is a grounder capable of translating logic programs provided by users into equivalent ground programs. The output of gringo can be piped into solver clasp [20], which then computes answer sets. System clingo internally couples gringo and clasp, thus, it takes care of both grounding and solving. In contrast to gringo outputting ground programs, clingo returns answer sets.

Usually logic programs are specified in one or more text files whose names are passed via the command line in an invocation of either gringo or clingo. We below provide a description of constructs belonging to the input language of gringo and clingo.

#### 3.1.1 Normal Programs and Integrity Constraints

Every logic program is constructed from terms. An overview of gringo terms is depicted in Figure 2. The most basic terms are integers, constants, and variables. Furthermore, there are some special variables and constants. An anonymous variable denoted by '\_' is simialar to a normal variable but each occurrence is treated like a different variable (intuitively a new unique variable name is substituted). Additionally, there are the two special constants '#supremum' and '#infimum' representing the largest and the smallest possible values, respectively, which behave essentially like constants. Finally, there are function symbols which are composed of other terms. A term that does not contain any (anonymous) variables is called a ground term. More complex terms involving arithmetics and other constructs are introduced later on.

Rules are defined as follows:

**Rule:** 
$$A_0 := -L_1, \ldots, L_n$$
.  
**Fact:**  $A_0$ .  
**Integrity Constraint:**  $:= -L_1, \ldots, L_n$ .

constant



variable



simpleterm



function



term



Figure 2: Terms

The head  $A_0$  of a fact or a rule is an *atom* of the same form as a function symbol or constant. Any  $L_j$  is a *literal* of the form A or not A for an atom A where the connective not corresponds to default negation. The set of literals  $\{L_1, \ldots, L_n\}$  is called the body of the rule. Facts have an empty body. Throughout this section we further extend the predicates that can be used in a rule including comparison predicates (Section 3.1.5) and aggregates (Section 3.1.10). Furthermore, gringo expects rules to be safe, i.e., all variables that appear in a rule have to appear in some positive literal (a literal not preceded by not) in the body. If a variable appears positively in some predicate, then we say that this predicate binds the variable.

Intuitively, the head of a rule has to be true whenever all its body literals are true. In ASP every atom needs some derivation, i.e., an atom cannot be true if there is no rule deriving it. This implies that only atoms appearing in some head can appear in answer sets. Furthermore, derivations<sup>1</sup> have to be acyclic, a feature that is important to model reachability. As a simple example, consider the program a := b. b := a. The only answer set to this program is the empty set. Adding either a. or b. to the program results in the answer set  $\{a, b\}$ . Finally, note that default negation is ignored when checking for acyclic derivations (we do not need a reason for an atom being false). Default negation can be used to express choices, e.g., the program a := not b. b := not a. has the two answer sets  $\{a\}$  and  $\{b\}$ . But in practice it is never needed to express choices this way. For example in the introductory example in Section 2 we used a cardinality constraint, which provides a much more readable way to introduce choices.

A fact has an empty body and thus its associated head predicate is always true and appears in all answer sets. On the other hand, integrity constraints eliminate answer set candidates. They are merely tests that discard unwanted answer sets. That is, there are no answer sets that satisfy all literals an integrity constraint. Elaborate examples on the usage of facts, rules, and integrity constraints are provided in Section 4.

#### 3.1.2 Classical Negation

In logic programs, connective not expresses default negation, that is, a literal not A is assumed to hold unless A is derived. In contrast, the classical (or strong) negation of some proposition holds if the complement of the proposition is derived [27]. Classical negation, indicated by symbol "–," is permitted in front of atoms. That is, if A is an atom, then -A is the complement of A. Semantically, -A is simply a new atom, with the additional condition that A and -A must not jointly hold. Observe that classical negation is merely a syntactic feature that can be implemented via integrity constraints whose effect is to eliminate any answer set candidate containing complementary atoms.

Example 3.1. Consider a logic program comprising the following facts:

```
1 bird(tux). penguin(tux).
2 bird(tweety). chicken(tweety).
3 flies(X) :- bird(X), not -flies(X).
4 -flies(X) :- bird(X), not flies(X).
5 -flies(X) :- penguin(X).
```

Logically, classical negation is reflected by (implicit) integrity constraints as follows:

By invoking gringo -t \ bird.lp flycn.lp the reader can observe that gringo indeed produces the integrity constraint in Line 7.

<sup>&</sup>lt;sup>1</sup>There are extensions like disjunctions that go beyond simple derivability and also require minimality w.r.t. a reduct. We do not cover the semantics of such constraints in this guide.

```
6 :- flies(tux), -flies(tux).
7 :- flies(tweety), -flies(tweety).
```

The program has two answer sets. One contains flies (tweety) and the other contains -flies (tweety). Let us now add a new fact to the program:

```
8 flies(tux).
```

There no longer is any answer set for our new program using classical negation. In fact, answer set candidates that contain both flies (tux) and -flies (tux) violate the integrity constraint in Line 6.

#### 3.1.3 Disjunction

Disjunctive logic programs permit connective "|" between atoms in rule heads. A disjunction is true if at least one of its atoms is true. Additionally, logic programs have to satisfy a minimality criterion, which we do not detail in this guide. The simple program  $a \mid b$ . has the two answer sets  $\{a\}$  and  $\{b\}$  but does not admit the answer set a, b because it is no minimal model.

In general, the use of disjunction however increases computational complexity [12]. This is why clingo<sup>2</sup> and solvers like assat [37], clasp [20], nomore++ [1], smodels [51], and smodels<sub>cc</sub> [56] do not work on disjunctive programs. Rather, claspD [8], cmodels [28, 35], or gnt [33] need to be used for solving a disjunctive program.<sup>3</sup> We thus suggest to use "choice constructs" (cf. Section 3.1.10) instead of disjunction, unless the latter is required for complexity reasons (see [13] for an implementation methodology in disjunctive ASP).

#### 3.1.4 Built-In Arithmetic Functions

gringo and clingo support a number of arithmetic functions that are evaluated during grounding. The following symbols are used for these functions: + (addition), – (subtraction, unary minus),  $\star$  (multiplication), / or #div (integer division), \ or #mod (modulo function),  $\star \star$  or #pow (exponentation),  $|\cdot|$  or #abs (absolute value), & (bitwise AND), ? (bitwise OR), ^ (bitwise exclusive OR), and ~ (bitwise complement).

Example 3.2. The usage of arithmetic functions is illustrated by the logic program:

1	left	(7).
2	right	(2).
3	plus	(L + R) := left(L), right(R).
4	minus	(L - R) := left(L), right(R).
5	uminus	( - R) :- right(R).
6	times	(L $\star$ R) :- left(L), right(R).
7	divide1	(L / R ) :- left(L), right(R).
8	divide2	<pre>(R #div L ) :- left(L), right(R).</pre>
9	divide2	(#div(R,L)) :- left(L), right(R).
10	modulo1	(L $\setminus$ R) :- left(L), right(R).
11	modulo2	(L $\#$ mod R ) :- left(L), right(R).
12	modulo3	(#mod(L,R)) := left(L), right(R).
13	absolute1	(  - R ) :- right(R).

The unique answer set of the program, obtained after evaluating all arithmetic functions, can be inspected by invoking: gringo -t arithf.lp

<sup>2</sup>Run as a monolithic system performing both grounding and solving.

<sup>3</sup>System dlv [34] also deals with disjunctive programs, but it uses a different syntax than presented here.

14	absolute2	(#abs(- R))	:-	right(R).
15	power1	(L ** R)	:- left(L),	right(R).
16	power2	(L #pow R )	:- left(L),	right(R).
17	power2	(#pow(L,R))	:- left(L),	right(R).
18	bitand	(L & R)	:- left(L),	right(R).
19	bitor	(L ? R)	:- left(L),	right(R).
20	bitxor	(L ^ R)	:- left(L),	right(R).
21	bitneg	( ~ R)	:-	right(R).

Note that variables L and R are instantiated to 7 and 2, respectively, before arithmetic evaluations. Consecutive and non-separative (e.g., before "(") spaces can also be dropped, while spaces after tokens # div and # mod are mandatory. Furthermore, the argument of function # abs, # div, and # mod must be enclosed in parentheses. The four bitwise functions apply to signed integers, using the two's complement of a negative integer.

Note that it is important that variables in the scope of an arithmetic function are not bound by a corresponding atom. For instance, the rule p(X) := p(X+1). is not safe but p(X-1) := p(X). is. Although, the latter might produce an infinite grounding and gringo not necessarily halts when given such an input.

#### 3.1.5 Built-In Comparison Predicates

The following built-in predicates permit term comparisons within the bodies of rules: == (equal), != (not equal), < (less than), <= (less than or equal), > (greater than), >= (greater than or equal).

Example 3.3. The usage of comparison predicates is illustrated by the logic program:

```
1
  num(1).
            num(2).
2
 eq (X,Y) :- X
                      Y, num(X), num(Y).
                   ==
3
  neq(X,Y) :- X
                   ! = Y, num(X), num(Y).
4
  lt (X,Y) :- X
                   <
                       Y, num(X), num(Y).
5
  leq(X, Y) := X
                   <=
                       Y, num(X), num(Y).
6
 qt (X,Y) :- X
                   >
                       Y, num(X), num(Y).
7
  qeq(X, Y) := X
                   \geq Y, num(X), num(Y).
8
 all(X,Y) := X-1 < X+Y, num(X), num(Y).
9 non(X,Y) :- X/X > Y * Y, num(X), num(Y).
```

The last two lines hint at the fact that arithmetic functions are evaluated before comparison predicates, so that the latter actually compare integers.

All comparison predicates can also be used with arbitrary ground terms, as in the next program:

```
1 sym(1). sym(a). sym(f(a)).
2 eq (X,Y) :- X == Y, sym(X), sym(Y).
3 neq(X,Y) :- X != Y, sym(X), sym(Y).
4 lt (X,Y) :- X < Y, sym(X), sym(Y).
5 leq(X,Y) :- X <= Y, sym(X), sym(Y).
6 gt (X,Y) :- X > Y, sym(X), sym(Y).
7 geq(X,Y) :- X >= Y, sym(X), sym(Y).
```

Integers are compared in the usual way and constants are ordered lexicographically. Function symbols are compared first using their arity. If the arity differs, then the name

As above, invoking: gringo -t symbc.lp yields the unique answer set of the program in terms of facts.

```
The unique answer set of the program is obtained via call:
gringo -t arithc.lp
```

of the function symbol is compared lexicographically. If again the name differs, then arguments are compared component wise. Finally, integers are always smaller than constants and constants are always smaller than function symbols.  $\hfill \Box$ 

**Note** that a built-in comparison predicate cannot bind variables, i.e., when checking whether a rule is safe, comparison predicates are not considered to be positive.

#### 3.1.6 Assignments

The built-in predicates := and = can be used in the body of a rule to unify a term on their right-hand side to a (non-ground) term or variable on its left-hand side, respectively.

**Example 3.4.** The next program demonstrates how terms can be assigned to variables:

Line 3 contains four assignments, where the right-hand sides directly or indirectly depend on X and Y. These two variables are bound in Line 5 via atoms of predicate num/1. Also observe the different usage and role of built-in comparison predicate ==.

**Example 3.5.** The second program demonstrates the usage of :=, which allows for terms on the left hand side:

```
1 sym(f(a,1,2)). sym(f(a,1,3)). sym(f(b,d)).
2 sym((a,1,2)). sym((a,1,3)). sym((b,d)).
```

```
4 unifyf(X) :- f(a,X,X+1) := F, sym(F).
5 unifyt(X) :- (a,X,X+1) := T, sym(T).
```

Here the term f (a, X, X+1) is unified with every function symbol provided by sym/1. Note the usage of X+1 in the term. gringo does not try to unify any term containing arithmetic but in this example X occurs also directly as second argument of the argument and can thus be unified with. The term X + 1 is merely a test that is deferred and checked later. For example, the fourth line is equivalent to:

6 unifyf(X) :- 
$$f(a, X, Y)$$
 := F, sym(F), Y == X + 1.

Note that assignments to some extent can bind variables. Of course cyclic assignments cannot bind variables. For example the rule p(X) := X = Y, Y = X. is rejected by gringo. Either X or Y has to be provided by some positive predicate in this case. Additionally, unification is restricted to ground terms on the right hand side of the assignment, that is, all variables on the right hand side have to be bound by some other predicate.

#### 3.1.7 Intervals

In Line 1 of Example 3.4, there are five facts num(k) over consecutive integers k. For a more compact representation, gringo and clingo support integer intervals of the

The unique answer set of the program is obtained via call: gringo -t assign.lp

The unique answer set of the program is obtained via call: gringo -t unify.lp

 $\Box$ 

form  $i \dots j$ , where i and j are integers. Such an interval represents each integer k such that  $i \le k \le j$ , and intervals are expanded during grounding.

**Example 3.6.** The next program makes use of integer intervals:

```
1 num(1..5).
2 top5(5..9).
3 top(9).
4 top5num(1..X-4,5..X) :- num(X-4..X), top5(1..5), top(X).
The facts in Line 1 and 2 are expanded as follows:
```

num(1). num(2). num(3). num(4). num(5). top5(5). top5(6). top5(7). top5(8). top5(9).

By instantiating X to 9, the rule in Line 4 becomes:

top5num(1..5,5..9) :- num(5..9), top5(1..5), top(9).

It is expanded to the cross product  $(1..5) \times (5..9) \times (5..9) \times (1..5)$  of intervals:

```
top5num(1,5) :- num(5), top5(1), top(9).
top5num(2,5) :- num(5), top5(1), top(9).
top5num(5,5) :- num(5), top5(1), top(9).
top5num(1,6) :- num(5), top5(1), top(9).
top5num(2,6) :- num(5), top5(1), top(9).
        : :
top5num(5,9) :- num(5), top5(1), top(9).
top5num(1,5) :- num(6), top5(1), top(9).
top5num(2,5) :- num(6), top5(1), top(9).
        : :
top5num(5,9) :- num(9), top5(1), top(9).
top5num(1,5) :- num(5), top5(2), top(9).
top5num(2,5) :- num(5), top5(2), top(9).
        :: :
top5num(5,9) :- num(9), top5(4), top(9).
top5num(1,5) :- num(5), top5(5), top(9).
top5num(2,5) :- num(5), top5(5), top(9).
top5num(5,9) :- num(5), top5(5), top(9).
top5num(1,5) :- num(6), top5(5), top(9).
top5num(2,5) :- num(6), top5(5), top(9).
        : :
top5num(5,9) :- num(9), top5(5), top(9).
```

Note that only the rules with num (5) and top5(5) in the body actually contribute to the unique answer set of the above program by deriving all atoms top5num (m, n) for  $1 \le m \le 5$  and  $5 \le n \le 9$ .

Again the unique answer set is obtained via call: gringo -t int.lp

**Note** that as with built-in arithmetic functions, an integer interval mentioning some variable (like X in Line 4 of Example 3.6) cannot be used to bind the variable.

#### 3.1.8 Conditions

Conditions allow for instantiating variables to collections of terms within a single rule. This is particularly useful for encoding conjunctions or disjunctions over arbitrarily many ground atoms as well as for the compact representation of aggregates (cf. Section 3.1.10). The symbol ":" is used to formulate conditions.

Example 3.7. The following program uses conditions in a rule body and in a rule head:

```
1 person(jane). person(john).
2 day(mon). day(tue). day(wed). day(thu). day(fri).
3 available(jane) :- not on(fri).
4 available(john) :- not on(mon), not on(wed).
5 meet :- available(X) : person(X).
6 on(X) : day(X) :- meet.
```

We are particularly interested in the rules in Line 5 and 6, instantiated as follows:

```
5 meet :- available(jane), available(john).
6 on(mon) | on(tue) | on(wed) | on(thu) | on(fri) :- meet.
```

The conjunction in Line 5 is obtained by replacing X in available (X) with all ground terms t such that person (t) holds, namely, t = jane and t = john. Furthermore, the condition in the head of the rule in Line 6 turns into a disjunction over all ground instances of on (X) where X is substituted by some term t such that day (t) holds. That is, conditions in the body and in the head of a rule are expanded to different basic language constructs.

Composite conditions can also be constructed via ":," as in the additional rules:

```
7 day(sat). day(sun).
8 weekend(sat). weekend(sun).
9 weekdays :- day(X) : day(X) : not weekend(X).
```

Observe that we may use the same atom, viz., day (X), both on the left-hand and on the right-hand side of ":." Furthermore, negative literals like not weekend (X) can occur on both sides of a condition. Note that literals on the right-hand side of a condition are connected conjunctively, that is, all of them must hold for ground instances of an atom in front of the condition. Thus, the instantiated rule in Line 8 looks as follows:

8 weekdays :- day(mon), day(tue), day(wed), day(thu), day(fri).

The atoms in the body of this rule follow from facts, so that the rule can be simplified to a fact weekdays. (as done by gringo).  $\Box$ 

Note that there are three important issues about the correct usage of conditions:

- 1. All predicates of atoms on the right-hand side of a condition must be either domain predicates, i.e., predicates that can be completely evaluated during grounding, or built-in, which is due to the fact that conditions are evaluated during grounding.
- 2. Any variable occurring within a condition is considered as *local*, that is, a condition cannot be used to bind variables outside the condition. In turn, variables outside conditions are *global*, and each variable within an atom in front of a condition must occur on the right-hand side or be global.

The reader can reproduce these ground rules by invoking: gringo -t cond.lp 3. Global variables take priority over local ones, that is, they are instantiated first. As a consequence, a local variable that also occurs globally is substituted by a term before the ground instances of a condition are determined. Hence, the names of local variables must be chosen with care, making sure that they do not accidentally match the names of global variables.

#### 3.1.9 Pooling

Symbol ";" allows for pooling alternative terms to be used as argument within an atom, thus, specifying rules more compactly. An atom written in the form p(..., X; Y, ...) abbreviates two options: p(..., X, ...) and p(..., Y, ...). Pooled arguments in any term of a rule body (or on the right-hand side of a condition) are expanded to a conjunction of the options within the same body (or within the same condition), while they are expanded to multiple rules (or multiple literals connected via ",") when occurring in the head (or in front of a condition).

**Example 3.8.** The following logic program makes use of pooling:

```
1 sym(a). sym(b).
2 num(1). num(2).
3 mix(A;B,M;N) :- sym(A;B), num(M;N), not -mix(M;N,A;B).
4 -mix(M;N,A;B) :- sym(A;B), num(M;N), not mix(A;B,M;N).
```

Let us consider instantiations of the rule in Line 3 obtained with substitution  $\{A \mapsto a, B \mapsto b, M \mapsto 1, N \mapsto 2\}$ . Note that mix/2 and -mix/2 each admit four options, corresponding to the cross product of  $\{a, b\}$  substituted for A and B, respectively, together with  $\{1, 2\}$  substituted for M and N. While the instances obtained for mix/2 give rise to four rules, the instances for -mix/2 jointly belong to the body. The (repeated) body also contains two instances each of sym/1 and of num/1. We thus get the rules:

```
mix(a,1) :- sym(a), sym(b), num(1), num(2), not -mix(1,a),
not -mix(1,b), not -mix(2,a), not -mix(2,b).
mix(a,2) :- sym(a), sym(b), num(1), num(2), not -mix(1,a),
not -mix(1,b), not -mix(2,a), not -mix(2,b).
mix(b,1) :- sym(a), sym(b), num(1), num(2), not -mix(1,a),
not -mix(1,b), not -mix(2,a), not -mix(2,b).
mix(b,2) :- sym(a), sym(b), num(1), num(2), not -mix(1,a),
not -mix(1,b), not -mix(2,a), not -mix(2,b).
```

Additionally, there is the ;; operator for pooling, which can only be used to separate arguments of predicates. This operator does not work on single terms but simply lists arguments of predicates. The rules for expanding the predicates are the same as for the ; operator.

**Example 3.9.** The following example show the difference between the ; and ; ; operator:

p(1,2). p(2,3).
 p(X,Z) :- p(X,Y;;Y,Z).
 q(X,Z) :- q(X,Y;Y,Z).

The second line is expanded into the following:

Simplified versions of these rules are produced via call: gringo -t sep.lp

Simplified versions of these rules are produced via call: gringo -t pool.lp

2 p(X,Z) := p(X,Y), p(Y,Z).

and the third line into:

$$3 p(X,Z) := p(X,Y,Z), p(X,Y,Z).$$

Clearly, the first variant is the desired expansion in this case to calculate the transitive closure. Both operators have their usages in different scenarios to keep the encoding more compact and readable.  $\hfill \Box$ 

#### 3.1.10 Aggregates

An aggregate is an operation on a multiset of weighted literals that evaluates to some value. In combination with comparisons, we can extract a truth value from an aggregate's evaluation, thus, obtaining an aggregate atom. We consider aggregate atoms of the form:

$$l \text{ op } [L_1 = w_1, \ldots, L_n = w_n] u$$

An aggregate has a lower bound l, an upper bound u, an operation op, and a multiset of literal  $L_i$  each assigned to a weight  $w_i$ . An aggregate is true if operation op applied to the multiset of weights of true literals is between the bounds (inclusive). Currently, gringo supports the aggregates #sum (the sum of weights), #min (the minimum weight), #max (the maximum weight), and #avg (the average of all weights<sup>4</sup>). Furthermore, there are three aggregates that are syntactically different. The first is the #count aggregate:

$$l # count \{ L_1, \ldots, L_n \} u$$

which basically are #sum aggregates with all weights set to one and duplicate true literals counted only once. Finally, there are the two parity aggregates:

 $\# even \{ L_1, ..., L_n \} \# odd \{ L_1, ..., L_n \}$ 

These aggregates are true if the number of different true literals is even or odd, respectively.

As regards syntactic representation, weight 1 is considered a default, so that  $L_i=1$  can simply be written as  $L_i$ . For instance, the following (multi)sets of (weighted) literals are the same when combined with any kind of aggregate operation and bounds:

```
[a=1, not b=1, c=2] and
[a, not b, c=2].
```

Furthermore, keyword #sum may be omitted, which in a sense makes #sum the default aggregate operation. In fact, the following aggregate atoms are synonyms:

2 #sum [a, not b, c=2] 3 and 2 [a, not b, c=2] 3.

By omitting keyword #sum, we obtain the same notation as the one of so-called "weight constraints" [51, 53], which are actually aggregate atoms whose operation is addition.

It is important to note that the (weighted) literals within an aggregate belong to a multiset. In particular, if there are multiple occurrences  $L=w_1, \ldots, L=w_k$  of a literal L, in combination with #min and #max, it is not the same like having  $L=w_1+\cdots+w_k$ . To see this, note that the program consisting of the facts:

<sup>&</sup>lt;sup>4</sup>The average aggregate over an empty set of weights is defined to be always true irrespective of any bounds.

2 #max [a=2]. 2 #min [a=2].

has  $\{a\}$  as its unique answer set, while there is no answer set for:

2 #max [a,a]. 2 #min [a,a].

If literals ought not to be repeated, we can use #count instead of #sum. Syntactically, #count requires curly instead of square brackets, and there must not be any weights within a #count aggregate. Regarding semantics, (l #count  $\{L_1, \ldots, L_n\} u$  reduces to  $(l \text{ sum } [L_1=1, \ldots, L_m=1] u)$ , where  $\{L_1, \ldots, L_m\} = \{L_i \mid 1 \le i \le n\}$  is obtained by dropping repeated literals. Of course, the use of l and u is optional also with #count. As an example, note that the next aggregate atoms express the same:

```
1 #sum [a=1, not b=1] 1 and
1 #count {a,a, not b, not b} 1.
```

Keyword #count can be omitted (like #sum), so that the following are synonyms:

1 #count {a, not b} 1 and 1 {a, not b} 1.

The last notation is similar to the one of so-called "cardinality constraints" [51, 53], which are aggregate atoms using counting as their operation.

After considering the syntax and semantics of ground aggregate atoms, we now turn our attention to non-ground aggregates. Regarding contained variables, an atom occurring in an aggregate behaves similar to an atom on the left-hand side of a condition (cf. Section 3.1.8). That is, any variable occurring within an aggregate is a priori local, and it must be bound via a variable of the same name that is global or that occurs on the right-hand side of a condition (with the atom containing the variable in front). As with local variables of conditions, global variables take priority during grounding, so that the names of local variables must be chosen with care to avoid accidental clashes. Beyond conditions (which are more or less the natural construct to use for instantiating variables within an aggregate), classical negation (cf. Section 3.1.2), built-in arithmetic functions (cf. Section 3.1.4), intervals (cf. Section 3.1.7), and pooling (cf. Section 3.1.9) can be incorporated as usual within aggregates, where intervals and pooling are expanded locally.<sup>5</sup> That is, an interval gives rise to multiple literals connected via "," within the same aggregate. The same applies to pooling in front of a condition, while it turns into a composite condition chained by ":" on the right-hand side. Finally, note that aggregates #sum, #count, #min, and #max without bounds are also permitted on the right-hand sides of assignments, but using this feature is only recommended for aggregates whose atoms belong to domain predicates because space blow-up can become a bottleneck otherwise. The following example, making exhaustive use of aggregates, nonetheless demonstrates this and other features.

**Example 3.10.** Consider a situation where an informatics student wants to enroll for a number of courses at the beginning of a new term. In the university calendar, eight courses are found eligible, and they are represented by the following facts:

```
1 course(1,1,5). course(1,2,5).
2 course(2,1,4). course(2,2,4).
3 course(3,1,6).
4 course(4,1,3).
```

<sup>&</sup>lt;sup>5</sup>Assignments (cf. Section 3.1.6) are permitted on the right-hand sides of conditions only.

```
      5
      course(5,1,4).
      course(5,4,4).

      6
      course(6,2,2).
      course(6,3,2).

      7
      course(7,2,4).
      course(7,3,4).
      course(7,4,4).

      8
      course(8,3,5).
      course(8,4,5).
```

In an instance of course/3, the first argument is a number identifying one of the eight courses, and the third argument provides the course's contact hours per week. The second argument stands for a subject area: 1 corresponding to "theoretical informatics," 2 to "practical informatics," 3 to "technical informatics," and 4 to "applied informatics." For instance, atom course(1, 2, 5) expresses that course 1 accounts for 5 contact hours per week that may be credited to subject area 2 ("practical informatics"). Observe that a single course is usually eligible for multiple subject areas.

After specifying the above facts, the student starts to provide personal constraints on the courses to enroll. The first condition is that 3 to 6 courses should be enrolled:

9 3 { enroll(C) : course(C,\_,\_) } 6.

Instantiating the above #count aggregate yields the following ground rule:

9 3 { enroll(1), enroll(2), enroll(3), enroll(4), enroll(5), enroll(6), enroll(7), enroll(8) } 6.

Observe that an instance of atom enroll (C) is included for each instantiation of C such that course(C, S, H) holds for some values of S and H. Duplicates resulting from distinct values for S are removed, thus, obtaining the above set of ground atoms. The next constraints of the student regard the subject areas of enrolled courses:

```
10 :- [ enroll(C) : course(C,_,_) ] 10.
11 :- 2 [ not enroll(C) : course(C,2,_) ].
12 :- 6 [ enroll(C) : course(C,3,_), enroll(C) : course(C,4,_) ].
```

Each of the three integrity constraints above contains a sum aggregate, using default weight 1 for literals. Recalling that #sum aggregates operate on multisets, duplicates are not removed. Thus, the integrity constraint in Line 10 is instantiated as follows:

```
10 :- [ enroll(1) = 1, enroll(1) = 1,
enroll(2) = 1, enroll(2) = 1,
enroll(3) = 1, enroll(3) = 1,
enroll(4) = 1, enroll(4) = 1, enroll(4) = 1,
enroll(5) = 1, enroll(5) = 1,
enroll(6) = 1, enroll(6) = 1,
enroll(7) = 1, enroll(7) = 1, enroll(7) = 1,
enroll(8) = 1, enroll(8) = 1 ] 10.
```

Note that courses 4 and 7 count three times because they are eligible for three subject areas, viz., there are three distinct instantiations for S in course(4, S, 3) and course(7, S, 4), respectively. Comparing the above ground instance, the meaning of the integrity constraint in Line 10 is that the number of eligible subject areas over all enrolled courses must be more than 10. Similarly, the integrity constraint in Line 11 expresses the requirement that at most one course of subject area 2 ("practical informatics") is not enrolled, while Line 12 stipulates that the enrolled courses amount to less than six nominations of subject area 3 ("technical informatics") or 4 ("applied informatics"). Also note that, given the facts in Line 1–8, we could equivalently have used count rather than sum in Line 11, but not in Line 10 and 12.

```
e sub-
```

```
The full ground program is ob-
tained by invoking:
gringo -t aggr.lp
```

The remaining constraints of the student deal with contact hours. To express them, we first introduce an auxiliary rule and a fact:

```
13 hours(C,H) :- course(C,S,H).
14 max hours(20).
```

The rule in Line 13 projects instances of course/3 to hours/2, thereby, dropping courses' subject areas. This is used to not consider the same course multiple times within the following integrity constraints:

```
15 :- not M-2 [ enroll(C) : hours(C,H) = H ] M, max_hours(M).
16 :- #min [ enroll(C) : hours(C,H) = H ] 2.
17 :- 6 #max [ enroll(C) : hours(C,H) = H ].
```

As Line 15 shows, we may use default negation via "not" in front of aggregate atoms, and bounds may be specified in terms of variables. In fact, by instantiating M to 20, we obtain the following ground instance of the integrity constraint in Line 15:

```
15 :- not 18 [ enroll(1) = 5, enroll(2) = 4,
enroll(3) = 6, enroll(4) = 3,
enroll(5) = 4, enroll(6) = 2,
enroll(7) = 4, enroll(8) = 5 ] 20.
```

The above integrity constraint states that the #sum of contact hours per week must lie in-between 18 and 20. Note that the #min and #max aggregates in Line 16 and 17, respectively, work on the same (multi)set of weighted literals as in Line 15. While the integrity constraint in Line 16 stipulates that any course to enroll must include more than 2 contact hours, the one in Line 17 prohibits enrolling for courses of 6 or more contact hours. Of course, the last two requirements could also be formulated as follows:

```
16 :- enroll(C), hours(C,H), H <= 2.
17 :- enroll(C), hours(C,H), H >= 6.
```

Finally, the following rules illustrate the use of aggregates within assignments:

```
18 courses(N) :- N = #count { enroll(C) : course(C,_,_) }.
19 hours(N) :- N = #sum [ enroll(C) : hours(C,H) = H ].
```

Note that the above aggregates have already been used in Line 9 and 15, respectively, where keywords #count and #sum have been omitted for convenience. These keywords can be dropped here too, and we merely include them to show the more verbose notations of #count and #sum aggregates. However, the usage of aggregates in the last two lines is different from before, as they now serve to assign an integer to a variable N. In this context, bounds are not permitted, and so none are provided in Line 18 and 19. The effect of these two lines is that the student can read off the number of courses to enroll and the amount of contact hours per week from instances of courses/1 and hours/1 belonging to an answer set. In fact, running clasp shows the student that a unique collection of 5 courses to enroll satisfies all requirements: the courses 1, 2, 4, 5, and 7, amounting to 20 contact hours per week.

Although the above program does not reflect this possibility, it should be noted that (as has been mentioned in Section 3.1.8) multiple literals may be connected via ":" in order to construct composite conditions within an aggregate. As before, the predicates of atoms on the right-hand side of such conditions must be either domain predicates or built-in. Furthermore, the usage of non-domain predicates within an aggregate on the right-hand side of an assignment (like enroll/1 in Line 18 and 19 above) is not recommended in general because the space blow-up may be significant.

To compute the unique answer set of the program, invoke: gringo aggr.lp | \ clasp -n 0 or alternatively: clingo -n 0 aggr.lp

#### 3.1.11 Optimization

Optimization statements extend the basic question of whether a set of atoms is an answer set to whether it is an optimal answer set. To support this reasoning mode, gringo and clingo adopt the optimization statements of lparse [53], indicated via keywords #maximize and #minimize. As an optimization statement does not admit a body, any (local) variable in it must also occur in an atom (over a domain or built-in predicate) on the right-hand side of a condition (cf. Section 3.1.8) within the optimization statement. In multiset notation (square brackets), weights may be provided as with #sum aggregates. In set notation (curly brackets), duplicates of literals are removed as with count aggregates. Additionally, priorities can be associated with each literal. A (ground) optimize statement has the form:

opt [ 
$$L_1 = w_1@p_1, \dots, L_n = w_n@p_n$$
 }  
opt {  $L_1@p_1, \dots, L_n@p_n$  }

where opt is either #maximize or #minimize,  $L_i$  are literals with associates (integer) weights  $w_i$  and (integer) priorities  $p_i$ .

The semantics of an optimization statement is intuitive: an answer set is *optimal* if the sum of weights (using 1 for unsupplied weights) of literals that hold is maximal or minimal, as required by the statement, among all answer sets of the given program. This definition is sufficient if a single optimization statement is specified along with a logic program. If different priorities occur in the program, then, depending on the type of optimize statement, answer sets whose sum of weights assigned to higher priorities is maximized or minimized, respectively.

Note that for compatibility with lparse, if multiple optimize statements are used, default priorities are assigned. The *n*-th statement gets priority n, thus, the later statements have higher priorities. We suggest that if you want to use more than one optimization statement, to always specify priorities to make the program more readable and order independent.

**Example 3.11.** To illustrate optimization, we consider a hotel booking situation where we want to choose one among five available hotels. The hotels are identified via numbers assigned in descending order of stars. Of course, the more stars a hotel has, the more it costs per night. As an ancillary information, we know that hotel 4 is located on a main street, which is why we expect its rooms to be noisy. This knowledge is specified in Line 1–5 of the following program:

```
1 1 { hotel(1..5) } 1.
2 star(1,5). star(2,4). star(3,3). star(4,3). star(5,2).
3 cost(1,170). cost(2,140). cost(3,90). cost(4,75). cost(5,60).
4 main_street(4).
5 noisy :- hotel(X), main_street(X).
6 #maximize [ hotel(X) : star(X,Y) = Y @ 1 ].
7 #minimize [ hotel(X) : cost(X,Y) : star(X,Z) = Y/Z @ 2 ].
8 #minimize { noisy @ 3 }.
```

Line 6–8 contribute optimization statements in inverse order of significance, according to which we want to choose the best hotel to book. The most significant optimization statement in Line 8 states that avoiding noise is our main priority. The secondary optimization criterion in Line 7 consists of minimizing the cost per star. Finally, the third optimization statement in Line 6 specifies that we want to maximize the number

of stars among hotels that are otherwise indistinguishable. The optimization statements in Line 6–8 are instantiated as follows:

If we now use clasp to compute an optimal answer set, we find that hotel 4 is not eligible because it implies noisy. Thus, hotel 3 and 5 remain as optimal w.r.t. the second most significant optimization statement in Line 7. This tie is broken via the least significant optimization statement in Line 6 because hotel 3 has one star more than hotel 5. We thus decide to book hotel 3 offering 3 stars to cost 90 per night.  $\Box$ 

3.1.12 Meta-Statements

After considering the language of logic programs, we now introduce features going beyond the contents of a program.

**Comments.** To keep records of the contents of a logic program, a logic program file may include comments. A comment until the end of a line is initiated by symbol "%," and a comment within one or over multiple lines is enclosed in "% \*" and "\* %." As an abstract example, consider:

```
logic program %* enclosed comment *% logic program
logic program % comment till end of line
logic program
%*
comment over multiple lines
*%
logic program
```

**Hiding Predicates.** Sometimes, one may be interested only in a subset of the atoms belonging to an answer set. In order to suppress the atoms of "irrelevant" predicates from the output, the #hide declarative can be used. The meanings of the following statements are indicated via accompanying comments:

```
#hide. % Suppress all atoms in output
#hide p/3. % Suppress all atoms of predicate p/3 in output
#hide p(X,Y) : q(X). % Supress p/3 if the condition holds
```

Note that for the conditionals on the right-hand side, the same conditions as described in Section 3.1.8 apply.

In order to selectively include the atoms of a certain predicate in the output, one may use the #show declarative. Here are some examples:

#show p/3. % Include all atoms of predicate p/3 in output
#show(X,Y) : q(X). % Include p/3 if the condition holds

A typical usage of #hide and #show is to hide all predicates via "#hide." and to selectively re-add atoms of certain predicates p/n to the output via "#show p/n."

The full ground program is obtained by invoking: gringo -t opt.lp

To compute the unique optimal answer set, invoke: gringo opt.lp | \ clasp -n 0 or alternatively: clingo -n 0 opt.lp **Constant Replacement.** Constants appearing in a logic program may actually be placeholders for concrete values to be provided by a user. An example of this is given in Section 4.1. Via the #const declarative, one may define a default value to be inserted for a constant. Such a default value can still be overridden via command line option --const (cf. Section 5.1). Syntactically, #const must be followed by an assignment having a (symbolic) constant on the left-hand side and a term on the right-hand side. Some exemplary constant declarations are:

 $\begin{aligned} &\# \text{const } x = 42. \\ &\# \text{const } y = f(x, h). \end{aligned}$ 

Note that (for efficiency reasons) constant declarations are order dependent. In the example above x would be replaced by 42 but when reversing the directives this would no longer be the case.

**Domain Declarations.** Usually, variable names are local to a rule, where they must be bound via appropriate atoms. This locality can be undermined by using #domain declarations that globally associate variable names to atoms. An associated atom is then simply added to the body of a rule in which such a predefined variable name occurs in. The following is a made-up example:

p(1,1). p(1,2).
#domain p(X,Y).
#domain p(Y,Z).
q(Z,X) :- not p(Z,X).

The above program is a priori not safe because variables X and Z are unbound in the last rule. However, as they belong to #domain declarations, gringo and clingo expand the last rule to:

q(Z,X) := p(X,Y), p(Y,Z), not p(Z,X).

Observe that the resulting program is safe.

**Note** that we suggest not to use domain statements because in ASP it is common to use very short variable names and using domain statements likely results in name clashes and undesired behavior.

**Compute Statements.** These statements are artifacts supported for backward compatibility. Although we strongly recommend to avoid compute statements, we now describe their syntax. A compute statement is of the form " $\#compute n\{...\}$ ." (the non-negative integer *n* is optional), where the " $\{...\}$ " part is similar to a #count aggregate. The meaning is that all literals contained in " $\{...\}$ " must hold w.r.t. answer sets that are to be computed, while *n* specifies a number of answer sets to compute. As clasp, clingo, and iclingo provide command line option --number (cf. Section 5.4) to specify how many answer sets are to be computed, they simply ignore *n*. Furthermore, the " $\{...\}$ " part can equivalently be expressed in terms of integrity constraints, as indicated in the comments provide along with the following example:

```
q(1;2).
{ p(1..5) }.
#compute 0 { p(X) : q(X) }. % :- 1 { not p(X) : q(X) }.
#compute { not p(X) : X=4..5 }. % :- 1 { p(X) : X=4..5 }.
```

Note that compute statement are not needed in general. The same behavior can be achieved by using integrity constraints. In fact, compute statements exist mainly for compatibility reasons with lparse. We suggest to not use them.

**External Statements.** External statements are used to mark certain atoms as external. This means that those atoms are not subject of simplification and consequently are not removed from the logic program. There are two kinds of external directives, global and local external statements.

Global external statements have the form #external predicate/arity. where predicate refers to the name of a predicate and arity to the arity of the predicate. They mark complete predicates irrespective of any arguments as external. This means that nothing is known about the predicate and hence it cannot be used for instantiation.

Example 3.12. Consider the following example:

```
#external q/1.
p(1). p(2).
r(X) :- q(X), p(X).
```

Here, the external predicate q/1 is not used for simplification of the problem and hence two ground rules (excluding facts) are printed.

Local external statements have the form #external predicate (: conditional)\*. where predicate is some non-ground predicate and conditional some conditional. In contrast to global external directives, local external statements precisely specify which atoms are external and hence can be used for instantiation.

Example 3.13. Again, consider a similar example:

```
#external q(X) : p(X).
p(1). p(2).
r(X) :- q(X).
```

Here, the external predicate q/1 is used to bind variable X, yielding the same rules as in the example above.

Furthermore, the lparse output[53] has been modified to include an additional table that stores a list of all external atoms. For compatibility, this table is only inserted if the program actually contains external directives. It contains the respective atom indices terminated by a zero and is inserted directly after lparse' compute statement.

**Example 3.14.** The following listing shows schematic example output of gringo when external statements are used:

0 0 B+ 0 B-

. . . 0 Ε 2 3 4 . . . 0 1

## 3.1.13 Integrated Scripting Language

Utilizing the scripting language Lua<sup>6</sup>, gringo's input language can be enriched by arbitrary arithmetical functions and implicit domains, answer sets can be intercepted and for example inserted into a database, or interactions between grounding and solving are possible when incrementally solving with iclingo. We do not give an introduction to Lua here (there are numerous tutorials on the web), but give some examples showing the capabilities of this integration.

**Example 3.15.** The first example shows basic Lua usage:

```
1
   #begin_lua
3
   function gcd(a, b)
4
      if a == 0 then return b
5
     else return gcd(b % a, a)
6
      end
7
   end
9
   function rng(a, b)
10
     r = {}
11
      for x = a, b do
12
        table.insert(r, x)
13
      end
14
      return r
15
   end
17
   #end_lua.
19 p(15,25).
20 gcd(X, Y, @gcd(X, Y)) := p(X, Y).
21 rng(X, Y, @rng(X, Y)) := p(X, Y).
```

In Line 3 we add a function that calculates the greatest common divisor of two numbers. This function is called in Line 20 and the result stored in predicate gcd/3. Note that Lua function calls look like function symbols but are preceded by "@". Regarding binding of variables, the same restrictions as with arithmetic in Section 3.1.4 apply.

To compute the unique answer set, invoke: gringo -t luaf.lp

<sup>&</sup>lt;sup>6</sup>http://www.lua.org

In Line 9 we add a function that emulates a range term. It returns a table containing all numbers in the interval [a,b]. The values in this table are then successively inserted for the call rng (X, Y). In fact, this function exactly behaves like a range term.

Val.NUM	Type identifier for gringo numbers.
Val.ID	Type identifier for gringo strings.
Val.FUNC	Type identifier for function symbols.
Val.SUP	Type identifier for gringo's #supremum.
Val.INF	Type identifier for gringo's #infimum.
<pre>Val.new(type[,value][,args])</pre>	Creates new ground terms (see Example 3.16).
Val.cmp(a,b)	Compares two ground gringo terms.
Val.type(a)	Returns the type of a term.
Val.name(f)	Returns the name of function symbol f.
Val.args(f)	Returns the arguments of function symbol f

Table 1: The Val meta-table.

**Example 3.16.** The second example shows how to create ground terms from within Lua:

```
1
  #begin_lua
 2 function f(x)
 3
     if
            Val.type(x) == Val.NUM then
 4
       return Val.new(Val.FUNC, { "num", x })
 5
     elseif Val.type(x) == Val.ID
                                   then
 6
       return Val.new(Val.FUNC, { "str", x })
 7
     elseif Val.type(x) == Val.FUNC then
 8
       local f = Val.new(Val.FUNC, x:name(), x:args())
9
       return Val.new(Val.FUNC, { "fun", f })
10
     elseif Val.type(x) == Val.INF then
11
       return Val.new(Val.FUNC, { "inf", Val.new(Val.INF) })
12
     elseif Val.type(x) == Val.SUP then
13
       return Val.new(Val.FUNC, { "sup", Val.new(Val.SUP) })
14
     end
15 end
17 function g(a,b)
18
     return Val.cmp(a, b)
19 end
21 function h(x)
22
     return '"' .. tostring(x) .. '"'
23 end
24 #end_lua.
26 p(1). p(a). p(\#supremum).
27 p(f(d(x), 1)). p(\#infimum).
29 type(X,Y) :- (X,Y) := @f(Z), p(Z).
```

To compute the unique answer set, invoke: gringo -t luav.lp

Function f in Line 2 returns a tuple whose first member is a string indicating the type of the argument and the second reconstructs the value passed to function f. Note that in Line 4, 6, 9, 11 and 13 the function Val.new is called. Its first argument indicates that a function symbol is to be created and the second argument passes the arguments of the function symbol. We do not give a name for the function symbol, thus a tuple is created (this is equivalent to passing the empty string as name). Similarly, gringo's other in-built ground terms are created. Finally, note that gringo integers and strings are directly mapped to the respective Lua types.

Assignment.begin(n,a)	Starts iteration over an atom with name n
	with arity a.
Assignment.next()	Advances to the next atom in the assign-
	ment and returns false if there is none.
Assignment.args()	The arguments of the current atom.
Assignment.isTrue()	The current atom is true.
Assignment.isFalse()	The current atom is false.
Assignment.isUndef()	The current atom is undefined.
Assignment.level()	The decision level on which the current
	atom has been assigned.

Table 2: The Assignment meta-table.

**Example 3.17.** The next example show some advanced usage also accessing clasp's truth assignment:

```
1
   #begin_lua
2
     local n
                 = 0
3
     local env = luasql.sqlite3()
4
     local conn = env:connect("test.sqlite3")
5
     conn:execute("CREATE TABLE IF NOT EXISTS test (x, y)")
6
     conn:execute("CREATE UNIQUE INDEX IF NOT EXISTS \
7
                    test_index ON test (x, y)")
9
     function query()
10
       local cur = conn:execute("SELECT * FROM test")
11
       local res = \{\}
12
       while true do
13
          local row = \{\}
14
          row = cur:fetch(row, "n")
15
         if row == nil then break end
16
          res[#res + 1] = Val.new(Val.FUNC, row)
17
       end
18
       cur:close()
19
       return res
20
     end
22
     function insert (name)
```

```
23
       Assignment.begin(name, 2)
24
       while Assignment.next() do
25
          if Assignment.isTrue() then
26
            local x = Assignment.args()[1]
27
            local y = Assignment.args()[2]
            local res = conn:execute("INSERT INTO test \
28
29
                                        VALUES(" ... x ... ", " ... y ... ")")
30
            if res \tilde{} = nil then n = n + 1 end
31
          end
32
        end
33
      end
35
      function onBeginStep() insert("p") end
36
      function onModel()
                              insert("q") end
37
                              print("inserted " .. n .. " values") end
      function onEndStep()
   #end_lua.
39
41
   p(1,1).
42 p(X+1,Y) := (X,Y) := @query().
43 \# odd { q(X, Y+1) : p(X, Y) }.
```

This example creates a database connection using SQLite<sup>7</sup>. The database connection is created Lines 3-4 using LuaSQL<sup>8</sup>. Initially, a new table test is created if it does not already exists (Lines 5-7). There are two functions to access this table. The first function query in Line 9 selects everything from the table and returns it in form of a Lua table, which is then used in the logic program to provide new instantiations of p/2in Line 42. The second function insert is a helper function that expects a predicate name and then inspects clingo's (or iclingo's) possibly partial assignment and inserts all true atoms into the database. It makes use of the Assignment meta-table. Next, note the three functions on BeginStep, on Model, and on EndStep in Line 35-37. The first function is called directly before solving but after preprocessing. At this point grounded facts are accessible via the Assignment meta-table (Table 2), we call insert to insert all ground instantiation of p/2 into the test table. The same is done in the onModel function whenever a model is found. At this point clingo's assignment is total and we insert all instantiation of q/2 into the database that are contained in the answer set. Finally, in function onEndStep we print the number of tuples inserted during grounding and after solving.

Note that the values from the answer set are inserted into the database. In fact the output of the program changes when it is consecutively called. Even more non-determinism can be added by using the option -rand-freq=1.0 to induce 100 percent random decisions.

Try to invoke this program multiple times using: clingo sql.lp

## 3.2 Input Language of iclingo

System iclingo [19] extends clingo by an *incremental* computation mode that incorporates both grounding and solving. Hence, its input language includes all constructs described in Section 3.1. In addition, iclingo deals with statements of the

<sup>&</sup>lt;sup>7</sup>http://www.sqlite.org/

<sup>&</sup>lt;sup>8</sup>http://www.keplerproject.org/luasql/

following form:

```
#base.
#cumulative constant.
#volatile constant.
```

Via "#base.," the subsequent part of a logic program is declared as static, that is, it is processed only once at the beginning of an incremental computation. In contrast, "#cumulative constant." and "#volatile constant." are used to declare a (symbolic) constant as a placeholder for incremental step numbers. In the parts of a logic program below a #cumulative statement, constant is in each step replaced with the current step number, and the resulting rules, facts, and integrity constraints are accumulated over a whole incremental computation. While the replacement of constant is similar, a logic program part below a #volatile statement is local to steps, that is, all rules, facts, and integrity constraints computed in one step are dismissed before the next incremental step. Note that the type of a logic program part (static, cumulative, or volatile) is determined by the last #base, #cumulative, or #volatile statement preceding it.

During an incremental computation, all static program parts are grounded first, while cumulative and volatile parts are grounded step-wise, replacing *constants* with successive step numbers starting from 1. Note that due to gringo's grounding algorithm, rules are not grounded twice using the same substitution of global variables (the incremental constant is treated like a global constant here). After a grounding step, clasp is usually invoked via an internal interface (like with clingo), and the incremental computation stops after a step in which at least one answer set has been found by clasp. This default behavior can be readapted via command line options (cf. Section 5.3). For obtaining a well-defined incremental computation result, it is important that (ground) head atoms within static, cumulative, and volatile program parts are distinct from each other, and they must also be different from step to step (see [19] for details). In Section 4.3, we provide a typical example in which these conditions naturally hold.

**Example 3.18.** For now, consider this simple Schur number example:

```
1 #base.
2 p(5).
3 #cumulative k.
4 1 { p(k,1..P) : p(P) } 1.
5 :- p(X,P), p(Y,P), X <= Y, p(X+Y,P).
6 :- not p(k,X) : X = 1..k, k <= P, p(P).</pre>
```

The Schur number n w.r.t. to a given number c is the largest integer such that the interval [1..n] can be partitioned into c sum-free sets. A set  $S = \{c_1, \ldots, c_k\}$  is sum-free if for each  $x_i$  and  $x_j$ , it holds that  $x_i + x_j \notin S$ .

In the base part in Line 2 we specify the number of partitions. Then, in the cumulative part in Line 4 each fresh integer k is assigned to exactly one partition. In Line 5 we check whether the guessed partition is sum-free. Note that we put this check in the cumulative part and incrementally extend it. The idea here is to keep as much constraints in the cumulative part because clasp applies nogood learning and only information learnt from the cumulative part can be kept among further solving steps; all information learnt from the volatile part has to be forgotten. Additionally, we use the comparison X <= Y, which helps to avoid grounding some redundant rules. Furthermore, note

To calculate the Schur number, invoke: iclingo inc.lp \ --istop=UNSAT The solving stops when the largest number has been found. that there appears no incremental constant in this rule, gringo's grounding algorithm makes sure that no ground instantiation of this rule is grounded twice just the new slice for the next step is instantiated. Finally, we break symmetries in Line 6, i.e., number one is always assigned to partition one, number two to partition two if it is not in partition one, and so on.

We use option --istop=UNSAT to solve as long as iclingo is able to find a solution, i.e., there is a valid partitioning. Once this is no longer possible, the ground-ing/solving process stops.

## 3.3 Input Language of clasp

Solver clasp [20] works on logic programs in lparse's output format [53]. This numerical format, which is not supposed to be human-readable, is output by gringo and can be piped into clasp. Such an invocation of clasp looks as follows:

gringo [ options | filenames ] | clasp [ number | options ]

Note that number may be provided to specify a maximum number of answer sets to be computed, where 0 makes clasp compute all answer sets. This maximum number can also be set via option --number or its abbreviation -n (cf. Section 5.4). By default, clasp computes one answer set (if it exists). If a logic program in lparse's output format has been stored in a file, it can be redirected into clasp as follows:<sup>9</sup>

clasp [ number | options ] < file</pre>

Via option --dimacs, clasp can also be instructed to compute models of a propositional formula in DIMACS/CNF format [7]. If such a formula is contained in file, then clasp can be invoked in the following way:

clasp [ number | options ] --dimacs < file

Finally, clasp may be used as a library, as done within clingo and iclingo.

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Finally, clasp may be used as a library, as done within clingo and iclingo.

 $<sup>^9</sup>$ The same is achieved by using option --file or its short form -f (cf. Section 5.4).

<sup>&</sup>lt;sup>10</sup>The same is achieved by using option --file or its short form -f (cf. Section 5.4).



Figure 3: A Directed Graph with Six Nodes and 17 Edges.

# 4 Examples

We exemplarily solve the following problems in ASP: *N*-Coloring (Section 4.1), Traveling Salesperson (Section 4.2), and Blocks-World Planning (Section 4.3). While the first problem could likewise be solved within neighboring paradigms, the second one requires checking reachability, something that is rather hard to encode in either Boolean Satisfiability [6] or Constraint Programming [47]. The third problem coming from the area of planning illustrates incremental solving with iclingo.

## 4.1 N-Coloring

As already mentioned in Section 2, it is custom in ASP to provide a *uniform* problem definition [39, 42, 50]. We follow this methodology and separate the encoding from an instance of the following problem: given a (directed) graph, decide whether each node can be assigned one of N colors such that any pair of adjacent nodes is colored differently. Note that this problem is NP-complete for  $N \ge 3$  (see, e.g., [44]), and thus it seems unlikely that a worst-case polynomial time algorithm can be found. In view of this, it is convenient to reduce the particular problem to a declarative problem solving paradigm like ASP, where efficient off-the-shelf tools like gringo and clasp are ready to solve the problem reasonably well. Such a reduction is now exemplified.

#### 4.1.1 Problem Instance

We consider directed graphs specified via facts over predicates node/1 and edge/2.<sup>11</sup> The graph shown in Figure 3 is represented by the following set of facts:

```
1 % Nodes
2 node(1..6).
3 % (Directed) Edges
4 edge(1,2;3;4). edge(2,4;5;6). edge(3,1;4;5).
5 edge(4,1;2). edge(5,3;4;6). edge(6,2;3;5).
```

Recall from Section 3.1 that "..." and ";" in the head expand to multiple rules, which are facts here. Thus, the instance contains six nodes and 17 directed edges.

<sup>&</sup>lt;sup>11</sup>Directedness is not an issue in N-Coloring, but we will reuse our directed example graph in Section 4.2.



Figure 4: A 3-Coloring for the Graph in Figure 3.

#### 4.1.2 Problem Encoding

We now proceed by encoding N-coloring via non-ground rules that are independent of particular instances. Typically, an encoding consists of a *Generate*, a *Define*, and a *Test* part [36]. As N-Coloring has a rather simple pattern, the following encoding does not contain any Define part:

```
1 % Default
2 #const n = 3.
3 % Generate
4 1 { color(X,1..n) } 1 :- node(X).
5 % Test
6 :- edge(X,Y), color(X,C), color(Y,C).
```

In Line 2, we use the #const declarative, described in Section 3.1.12, to install 3 as default value for constant n that is to be replaced with the number N of colors. (The default value can be overridden by invoking gringo with option -const n=N.) The Generate rule in Line 4 makes use of the count aggregate (cf. Section 3.1.10). For our example graph and 1 substituted for X, we obtain the following ground rule:

1 { color(1,1), color(1,2), color(1,3) } 1.

Note that node (1) has been removed from the body, as it is derived via a corresponding fact, and similar ground instances are obtained for the other nodes 2 to 6. Furthermore, for each instance of edge/2, we obtain n ground instances of the integrity constraint in Line 6, prohibiting that the same color C is assigned to the adjacent nodes. Given n=3, we get the following ground instances due to edge (1, 2):

```
:- color(1,1), color(2,1).
:- color(1,2), color(2,2).
:- color(1,3), color(2,3).
```

Again note that edge (1, 2), derived via a fact, has been removed from the body.

#### 4.1.3 Problem Solution

Provided that a given graph is colorable with n colors, a solution can be read off an answer set of the program consisting of the instance and the encoding. For the graph in Figure 3, the following answer set can be computed:



The full ground program is obtained by invoking: gringo -t \ color.lp graph.lp



Figure 5: The Graph from Figure 3 along with Edge Costs.

```
Answer: 1
... color(1,2) color(2,1) color(3,1) \
color(4,3) color(5,2) color(6,3)
```

Note that we have omitted the six instances of node/1 and the 17 instances of edge/2 in order to emphasize the actual solution, which is depicted in Figure 4. Such output projection can also be specified within a logic program file by using the declaratives #hide and #show, described in Section 3.1.12.

## 4.2 Traveling Salesperson

We now consider the well-known Traveling Salesperson Problem (TSP), where the task is to decide whether there is a round trip that visits each node in a graph exactly once (viz., a Hamiltonian cycle) and whose accumulated edge costs must not exceed some budget B. We tackle a slightly more general variant of the problem by not a priori fixing B to any integer. Rather, we want to compute a minimum budget B along with a round trip of cost B. This problem is FP<sup>NP</sup>-complete (cf. [44]), that is, it can be solved with a polynomial number of queries to an NP-oracle. As with N-Coloring, we provide a uniform problem definition by separating the encoding from instances.

#### 4.2.1 Problem Instance

We reuse graph specifications in terms of predicates node/1 and edge/2 as in Section 4.1.1. In addition, facts over predicate cost/3 are used to define edge costs:

```
% Edge Costs
1
2
  cost(1,2,2). cost(1,3,3). cost(1,4,1).
3
  cost(2,4,2).
                cost(2,5,2). cost(2,6,4).
4
                cost(3,4,2).
  cost(3, 1, 3).
                              cost(3,5,2).
5
  cost(4,1,1).
                cost(4,2,2).
 cost(5,3,2).
                cost(5,4,2).
6
                              cost(5,6,1).
                cost(6,3,3).
7
 cost(6,2,4).
                              cost(6,5,1).
```

Figure 5 shows the (directed) graph from Figure 3 along with the associated edge costs. Symmetric edges have the same costs here, but differing costs would also be possible.

#### 4.2.2 Problem Encoding

The first subproblem consists of describing a Hamiltonian cycle, constituting a candidate for a minimum-cost round trip. Using the Generate-Define-Test pattern [36], we encode this subproblem via the following non-ground rules:

```
1
   % Generate
2
  1 { cycle(X,Y) : edge(X,Y) } 1 :- node(X).
3
  1 { cycle(X,Y) : edge(X,Y) } 1 :- node(Y).
4
  % Define
5
   reached(Y) :- cycle(1,Y).
6
  reached(Y) :- cycle(X,Y), reached(X).
7
   % Test
8
  :- node(Y), not reached(Y).
9
  % Display
10 #hide.
11 #show cycle/2.
```

The Generate rules in Line 2 and 3 assert that every node must have exactly one outgoing and exactly one incoming edge, respectively, belonging to the cycle. By inserting the available edges, for node 1, Line 2 and 3 are grounded as follows:

```
1 { cycle(1,2), cycle(1,3), cycle(1,4) } 1.
1 { cycle(3,1), cycle(4,1) } 1.
```

Observe that the first rule groups all outgoing edges of node 1, while the second one does the same for incoming edges. We proceed by considering the Define rules in Line 5 and 6, which recursively check whether nodes are reached by a cycle candidate produced via the Generate part. Note that the rule in Line 5 builds on the assumption that the cycle "starts" at node 1, that is, any successor Y of 1 is reached by the cycle. The second rule in Line 6 states that, from a reached node X, an adjacent node Y can be reached via a further edge in the cycle. Note that this definition admits positive recursion among the ground instances of reached/1, in which case a ground program is called non-tight [15, 16]. The "correct" treatment of (positive) recursion is a particular feature of answer set semantics, which is hard to mimic in either Boolean Satisfiability [6] or Constraint Programming [47]. In our present problem, this feature makes sure that all nodes are reached by a global cycle from node 1, thus, excluding isolated subcycles. In fact, the Test in Line 8 stipulates that every node in the given graph is reached, that is, the instances of cycle/2 in an answer set must be the edges of a Hamiltonian cycle. Finally, the additional Display part in Line 10 and 11 states that answer sets should be projected to instances of cycle/2, as only they describe a solution. We have so far not considered edge costs, and answer sets for the above part of the encoding correspond to Hamiltonian cycles, that is, candidates for a minimum-cost round trip.

In order to minimize costs, we add the following optimization statement:

```
13 % Optimize
```

14 minimize [ cycle(X,Y) : cost(X,Y,C) = C ].

Here, edges belonging to the cycle are weighted according to their costs. After grounding, the minimization in Line 14 ranges over 17 instances of cycle/2, one for each weighted edge in Figure 5. The full ground program is obtained by invoking: gringo -t \ ham.lp min.lp \ costs.lp graph.lp

To compute the six Hamiltonian cycles for the graph in Figure 3, invoke: gringo \ ham.lp graph.lp | \ clasp -n 0 or alternatively: clingo -n 0 \ ham.lp graph.lp



Figure 6: A Minimum-Cost Round Trip.

#### 4.2.3 Problem Solution

Finally, we explain how the unique minimum-cost round trip (depicted in Figure 6) can be computed. The catch is that we are now interested in optimal answer sets, rather than in arbitrary ones. In order to determine the optimum, we can start by gradually decreasing the costs associated to answer sets until we cannot find a strictly better one anymore. clasp (or clingo) enumerates successively better answer sets w.r.t. the provided optimization statements (cf. Section 3.1.11). Any answer set is printed as soon as it has been computed, and the last one is optimal. If there are multiple optimal answer sets, an arbitrary one among them is computed. For the graph in Figure 5, the optimal answer set (cf. Figure 6) is unique, and its computation can proceed as follows:

```
Answer: 1
cycle(1,3) cycle(2,4) cycle(3,5) \
cycle(4,1) cycle(5,6) cycle(6,2)
Optimization: 13
Answer: 2
cycle(1,2) cycle(2,5) cycle(3,4) \
cycle(4,1) cycle(5,6) cycle(6,3)
Optimization: 11
```

Given that no answer is obtained after the second one, we know that 11 is the optimum value, but there might be more optimal answer sets that have not been computed yet. In order to find them too, we can use the following command line options of clasp (cf. Section 5.4): "--opt-value=11" in order to initialize the optimum and "--opt-all" to compute also equitable (rather than only strictly better) answer sets. After obtaining only the second answer given above, we are sure that this is the unique optimal answer set, whose associated edge costs (cf. Figure 6) correspond to the reported optimization value 11. Note that, with #maximize statements in the input, this correlation might be less straightforward because they are compiled into #minimize statements in the process of generating lparse's output format [53]. Furthermore, if there are multiple optimization statements, clasp (or clingo) will report separate optimization values ordered by significance. Finally, the two-stage invocation scheme exercised above, first determining the optimum and afterwards all (and only) optimal answer sets, is recommended for general use. Otherwise, if using option "--opt-all" right away without knowing the optimum, one risks the enumer-



```
The full invocation is:

gringo \

ham.lp min.lp \

costs.lp graph | \

clasp -n 0 \

--opt-value=11 \

--opt-all

or alternatively:

clingo -n 0 \

--opt-value=11 \

--opt-all \

ham.lp min.lp \

costs.lp graph
```

ation of plenty suboptimal answer sets. We also invite everyone to explore command line option "--opt-restart" (cf. Section 5.4) in order to see whether it improves search efficiency on optimization problems.

## 4.3 Blocks-World Planning

The Blocks-World is a well-known planning domain, where finding *shortest* plans has received particular attention [32]. In an eventually propositional formalism like ASP, a bound on the plan length must be fixed before search can proceed. This is usually accomplished by including some constant t in an encoding, which is then replaced with the actual bound during grounding. Of course, if the length of a shortest plan is unknown, an ASP system must repeatedly be queried while varying the bound. With a traditional ASP system, processing the same planning problem with a different bound involves grounding and solving from scratch. In order to reduce such redundancies, the incremental ASP system iclingo[19] can gradually increase a bound, doing only the necessary additions in each step. Note that planning is a natural application domain for iclingo, but other problems including some mutable bound can be addressed too, thus, iclingo is not a specialized planning system. However, we use Blocks-World Planning to illustrate the exploitation of iclingo's incremental computation mode.

#### 4.3.1 Problem Instance

As with the other two problems above, an instance is given by a set of facts, here, over predicates block/1 (declaring blocks), init/1 (defining the initial state), and goal/1 (specifying the goal state). A well-known Blocks-World instance is described by:<sup>12</sup>

```
1
   % Sussman Anomaly
 2
   0
 3 block(b0).
 4
   block(b1).
 5
   block(b2).
 6
   8
 7
   % initial state:
   00
 8
9
   8
       2
10
   %
       0 1
11
   %
      _____
12
   %
13
   init(on(b1,table)).
14
   init(on(b2,b0)).
15
   init(on(b0,table)).
16
   8
17
   % goal state:
18
   90
19
       2
   %
20
   00
       1
21
   %
       0
22
   0
```

<sup>12</sup>Blocks-World instances world*i*.lp for  $i \in \{0, 1, 2, 3, 4\}$  are adaptations of the instances provided at [14].

23 % 24 goal(on(b1,b0)).

25 goal(on(b2,b1)).

26 goal(on(b0,table)).

Note that the facts in Line 13–15 and 24–26 specify the initial and the goal state depicted in Line 9-11 and 19–22, respectively. We here use (uninterpreted) function on/2 to illustrate another important feature available in gringo, clingo, and iclingo, namely, the possibility of instantiating variables to compound terms.

## 4.3.2 Problem Encoding

Our Blocks-World Planning encoding for iclingo makes use of declaratives #base, #cumulative, and #volatile, separating the encoding into a static, a cumulative, and a volatile (query) part. Each of them can be further refined into Generate, Define, Test, and Display constituents, as indicated in the comments below:

```
1
   #base.
2 % Define
3 location(table).
4 location(X) :- block(X).
5 holds (F, 0) :- init (F).
6 %
7
   #cumulative t.
8 % Generate
  1 { move(X,Y,t) : block(X) : location(Y) : X != Y } 1.
9
10 % Test
11
  :- move(X,Y,t),
12
      1 { holds(on(A,X),t-1),
13
           holds (on(B, Y), t-1) : B != X : Y != table }.
14 % Define
15 holds (on(X, Y), t) := move(X, Y, t).
   holds (on(X,Z),t) :- holds (on(X,Z),t-1),
16
17
                         \{ move(X, Y, t) : Y != Z \} 0.
18
   8
19
   #volatile t.
20 % Test
21 :- goal(F), not holds(F,t).
22 %
23 #base.
24 % Display
25 #hide.
26 \text{ #show move}/3.
```

In the initial #base part (Line 1–5), we define blocks and constant table as instances of predicate location/1. Moreover, we use instances of init/1 to initialize predicate holds/2 for step 0, thus, defining the conditions before the first incremental step. Note that variable F is instantiated to compound terms over function on/2.

The #cumulative statement in Line 7 declares constant t as a placeholder for step numbers in the cumulative encoding part below. Here, the Generate rule in Line 9 states that exactly one block X must be moved to a location Y (different from X) at each step t. The integrity constraint in Line 11–13 is used to test whether moving block X

```
38
```

to location Y is possible at step t by denying move (X, Y, t) to hold if there is either some block A on X or some block B distinct from X on Y (this condition is only checked if Y is a block, viz., different from constant table) at step t-1. Finally, the Define rule in Line 15 propagates a move to the state at step t, while the rule in Line 16–17 states that a block X stays on a location Z if it is not moved to any other location Y.

The #volatile statement in Line 19 declares the next part as a query depending on step number t, but not accumulated over successive steps. In fact, the integrity constraint in Line 21 tests whether goal conditions are satisfied at step t.

Our incremental encoding concludes with a second #base part, as specified in Line 23. Note that, for the meta-statements with Display functionality (Line 25–26), it is actually unimportant whether they belong to a static, cumulative, or volatile program part, as answer sets are projected (to instances of move/3) in either case. However, by ending the encoding file with a #base statement, we make sure that the contents of a concatenated instance file is included in the static program part. This is also the default of iclingo (as well as of gringo and clingo that can be used for non-incremental computations).

Finally, let us stress important prerequisites for obtaining a well-defined incremental computation result from iclingo. First, the ground instances of head atoms of rules in the static, cumulative, and volatile program part must be pairwisely disjoint. Furthermore, ground instances of head atoms in the volatile part must not occur in the static and cumulative parts, and those of the cumulative part must not be used in the static part. Finally, ground instances of head atoms in either the cumulative or the volatile part must be different for each pair of distinct steps. This is the case for our encoding because both atoms over move/3 and those over holds/2 include t as an argument in the heads of the rules in Line 9, 15, and 16–17. As the smallest step number to replace t with is 1, there also is no clash with the ground atoms over holds/2 obtained from the head of the static rule in Line 5. Further details on the sketched requirements and their formal background can be found in [19]. Arguably, many problems including some mutable bound can be encoded such that the prerequisites apply. Some attention should of course be spent on putting rules into the right program parts.

#### 4.3.3 Problem Solution

We can now use iclingo to *incrementally* compute the shortest sequence of moves that brings us from the initial to the goal state depicted in the instance in Section 4.3.1:

```
Answer: 1
move(b2,table,1) move(b1,b0,2) move(b2,b1,3)
```

This unique answer set tells us that the given problem instance can be solved by moving block b2 to the table in order to then put b1 on top of b0 and finally b2 on top of b1. This solution is computed by iclingo in three grounding and solving steps, where, starting from the #base program, constant t is successively replaced with step numbers 1, 2, and 3 in the #cumulative and in the #volatile part. While the query postulated in the #volatile part cannot be fulfilled in steps 1 and 2, iclingo stops its incremental computation after finding an answer set in step 3. The scheme of iterating steps until finding at least one answer set is the default behavior of iclingo, which can be customized via command line options (cf. Section 5.3).

Finally, let us describe how solutions obtained via an incremental computation can be computed in the standard way, that is, in a single pass. To this end, the step number can be fixed to some n via option "--ifixed=n" (cf. Section 5.1), enabling

For non-incremental solving, invoke: gringo --ifixed=n \ blocks.lp \ world0.lp |\ clasp -n 0 or alternatively: clingo -n 0 \ --ifixed=n \ blocks.lp world0.lp

dealt with internally iclingo
at a step n, invoke:
 gringo -t \
 --ifixed=n \
 blocks.lp world0.lp
Furthermore you can try:
 world1.lp, world2.lp,
 world3.lp, world4.lp.

To observe the ground program

To this end, invoke: iclingo -n 0 \ blocks.lp world0.lp Furthermore you can try: world1.lp, world2.lp, world3.lp, world4.lp. gringo or clingo to generate the ground program present inside iclingo at step n. Note that #volatile parts are here only instantiated for the final step n, while #cumulative rules are added for all steps 1, ..., n. Option "--ifixed=n" can be useful for investigating the contents of a ground program dealt with at step n or for using an external solver (other than clasp). In the latter case, repeated invocations with varying n are required if the bound of an optimal solution is a priori unknown.

# **5** Command Line Options

In this section, we briefly describe the meanings of command line options supported by gringo (Section 5.1), clingo (Section 5.2), iclingo (Section 5.3), and clasp (Section 5.4). Each of these tools display their available options when invoked with flag -help or -h.<sup>13</sup> The approach of distinguishing long options, starting with "--," and short ones of the form "-1," where 1 is a letter, follows the GNU Coding Standards [29]. For obvious reasons, short forms are made available only for the most common (long) options. Some options, also called flags, do not take any argument, while others require arguments. An argument arg is provided to a (long) option opt by writing "--opt=arg" or "--opt arg," while only "-1 arg" is accepted for a short option 1. For each command line option, we below indicate whether it requires an argument, and if so, we also describe its meaning.

## 5.1 gringo Options

An abstract invocation of gringo looks as follows:

gringo [ options | filenames ]

Note that options and filenames do not need to be passed to gringo in any particular order. If neither a filename nor an option that makes gringo exit (see below) is provided, gringo reads from the standard input. In the following, we list and describe the options accepted by gringo along with their particular arguments (if required):

```
--help,-h
```

Print help information and exit.

```
--version,-v
```

Print version information and exit.

--verbose[=n],-V

Print additional (progress) information during computation. Verbosity level one and two are currently not used by gringo. Level three prints internal representations of rules during grounding. (This may be used to identify either semantic errors in an input program or performance bottlenecks.)

```
--const,-c c=t
```

Replace occurrences (in the input program) of a constant c with a term t.

--text,-t

Output ground program in (human-readable) text format.

<sup>&</sup>lt;sup>13</sup>Note that our description of command line options is based on Version 3.0.x of gringo, clingo, and iclingo as well as Version 1.3.x of clasp. While it is rather unlikely that command line options will disappear in future versions, additional ones might be introduced. We will try to keep this document up-to-date, but checking the help information shipped with a new version is always a good idea.

#### --reify

Output ground program in form of facts. (These facts can then be used to, i.e., write a meta-interpreter.)

#### --lparse,-l

Output ground program in lparse's numerical format [53].

#### --ground, -g

Enable lightweight mode for processing a ground input program. (This option is recommended to omit unnecessary overhead if the input program is already ground, but it leads to a syntax error (cf. Section 6.1) otherwise.)

### --ifixed=n

Use *n* as fix step number if the input program contains #cumulative or #volatile statements. (This option permits the handling of programs written for iclingo in a traditional single pass computation.)

#### --ibase

Process only the static part (that can be initiated by a #base statement) of an input program. (This option may be used to investigate the basic setting of a problem including some mutable bound.)

#### --dep-graph=filename

This option can be used to get the dependency graph in from of a dot<sup>14</sup> file of the program.

#### --shift

Removes disjunction by shifting (see [27]).

The default command line when invoking gringo is as follows:

gringo --lparse

That is, gringo usually outputs a ground program in lparse's numerical format, dealt with by various ASP solvers [20, 51, 37].

## 5.2 clingo Options

ASP system clingo combines grounder gringo and solver clasp via an internal interface. An abstract invocation of clingo looks as follows:

clingo [ number | options | filenames ]

A numerical argument is permitted for backward compatibility to the usage of solver smodels [51], where it specifies the maximum number of answer sets to be computed (0 standing for all answer sets). As with gringo, a number, options, and filenames do not need to be passed to clingo in any particular order. Given that clingo combines gringo and clasp, it accepts all options described in the previous section and in Section 5.4. In particular, (long) options --help and --version make clingo print the desired information and exit, while --text, --lparse, and --reify instruct clingo to output a ground program (rather than solving it) like gringo. If neither a filename nor an option that makes clingo exit (see Section 5.1) is provided, clingo reads from the standard input. Beyond the options described in Section 5.1 and 5.4, clingo has a single additional option:

<sup>&</sup>lt;sup>14</sup>http://www.graphviz.org/

#### --clasp

Run clingo as a plain solver (using embedded clasp).

Finally, the default command line when invoking clingo consists of all clasp defaults (cf. Section 5.4).

## 5.3 iclingo Options

Incremental ASP system iclingo extends clingo by interleaving grounding and solving for problems including a mutable bound. An abstract invocation of iclingo is as with clingo:

```
iclingo [ number | options | filenames ]
```

The external behavior of iclingo is similar to clingo, described in the previous section, except for the fact that option --ifixed is ignored by iclingo if not run as a grounder (via one of (long) options --text, --lparse, or --reify). However, option --clingo (see below) may be used to let iclingo work like clingo. The additional options of iclingo focus on customizing incremental computations:

#### --istats

Print statistic information for each incremental solving step.

--imin=n

Perform at least n incremental solving steps before termination. (This may be used to force steps regardless of the termination condition set via -istop.)

--imax=n

Perform at most n incremental solving steps before termination. (This may be used to limit steps regardless of the termination condition set via --istop.)

#### --istop=SAT|UNSAT

Terminate after an incremental solving step in which some (SAT) or no (UNSAT) answer set has been found.

--iquery=n

Start with incremental solving at step number *n*. (This may be used to skip some solving steps, still accumulating static and cumulative rules for these steps.)

#### --ilearnt=keep|forget

Maintain (keep) or delete (forget) learnt constraints in-between incremental solving steps. (This option configures the behavior of embedded clasp.)

## --iheuristic=keep|forget

Maintain (keep) or delete (forget) heuristic information in-between incremental solving steps. (This option configures the behavior of embedded clasp.)

#### --clingo

Run iclingo as a non-incremental ASP system (like clingo).

As with clingo, the default command line when invoking iclingo consists of all clasp defaults, explained in the next section, along with --istop=SAT, --iquery=1, --ilearnt=keep, and --iheuristic=forget. That is, incremental solving starts at step number 1 and stops after a step in which some answer set has been found. In-between incremental solving steps, embedded clasp maintains learnt constraints but deletes heuristic information.

## 5.4 clasp Options

Stand-alone clasp [20] is a solver for ground programs in lparse's numerical format [53]. Beyond that, it can also be used as a SAT (on a simplified version of DIMAC-S/CNF format<sup>15</sup>), or PB solver (on OPB format<sup>16</sup>). An abstract invocation of clasp looks as follows:

```
clasp [ number | options | filename ]
```

As with clingo and iclingo, a numerical argument specifies the maximum number of answer sets to be computed, where 0 stands for all answer sets. (The number of requested answer sets can likewise be set via long option --number or its short form -n.) If neither a filename nor an option that makes clasp exit (see below) is provided, clasp reads from the standard input.<sup>17</sup> In fact, it is typical to use clasp in a pipe with gringo in the following way:

```
gringo [ ... ] | clasp [ ... ]
```

In such a pipe, gringo instantiates an input program and outputs the ground rules in lparse's numerical format, which is then consumed by clasp that computes and outputs answer sets. Note that clasp offers plenty of options to configure its behavior. We thus categorize them according to their functionalities in the following description.

#### 5.4.1 General Options

We below group general options of clasp, used to configure its global behavior.

--help,-h

Print help information and exit.

--version,-v

Print version information and exit.

--pre

Run ASP preprocessor then print preprocessed input program and exit.

#### --verbose[=n],-V

Configure printing of (progress) information during computation. Argument n = 0 disables progress information, while n = 1 prints basic, and n = 2 extended information.

--stats

Print (extended) statistic information before termination.

--number,-n n

Compute at most *n* answer sets, n = 0 standing for compute all answer sets.

#### --quiet,-q

Do not print computed answer sets. (This is useful for benchmarking.)

#### --time-limit=t

Force termination after t seconds.

<sup>15</sup>http://www.satcompetition.org/2009/format-benchmarks2009.html

<sup>&</sup>lt;sup>16</sup>http://www.cril.univ-artois.fr/PB09/solver\_req.html

 $<sup>^{17}</sup>$ In earlier versions of clasp filenames had to be given via long option --file or its short form -f.

#### --search-limit=n,m

Force termination after either n conflicts or m restarts.

#### --seed=n

Use seed *n* (rather than 1) for random number generator.

#### --solution-recording

Switch from backtrack-based enumeration [21] to enumeration based on solution recording. Note that this mode is prone to blow up in space in view of an exponential number of solutions in the worst case.

#### --restart-on-model

Restart the search from scratch (rather than doing enumeration [21]) after finding an answer set. Note: This mode implies solution recording and hence has the same caveat. It is mainly useful when searching for an optimal solution because, first, restarting may greatly speed up finding the optimum, and second, only the current optimum needs to be recorded.

#### --project

Project answer sets to named atoms and only enumerate unique projected solutions [25].

#### --brave

Compute the brave consequences (union of all answer sets) of a logic program.

#### --cautious

Compute the cautious consequences (intersection of all answer sets) of a logic program.

#### --opt-all

Compute all optimal answer sets (cf. Section 3.1.11). (This is implemented by enumerating [21] answer sets that are not worse than the best one found so far.)

#### --opt-value=n1[, n2, n3...]

Initialize objective function(s) to minimize with n1[, n2, n3...].

#### --opt-ignore

Ignore any optimize statements of a logic program during computation.

#### --opt-heu

Consider optimize statements in heuristics.

#### --supp-models

Compute supported models [2] (rather than answer sets).

#### --trans-ext=all|choice|weight|dynamic|no

Compile extended rules [51] into normal rules of form (3.1.1). Arguments choice and weight state that all "choice rules" or all "weight rules," respectively, are to be compiled into normal rules, while all means that both and no that none of them are subject to compilation. If argument dynamic is given, clasp heuristically decides whether or not to compile individual "weight rules".

#### --eq=*n*

Run equivalence reasoning [23] for *n* iterations, n = -1 and n = 0 standing for run to fixpoint or do not run equivalence reasoning, respectively.

### --backprop

Enable backpropagation in ASP-preprocessing.

### --sat-prepro=yes|no|n1[,n2,n3]

Run *SatElite*-like preprocessing [10] for at most n1 iterations (n1 = -1 standing for run to fixpoint), using cutoff n2 for variable elimination (n2 = -1 standing for no cutoff), and for no longer than n3 seconds (n3 = -1 standing for no time limit). Arguments yes and no mean n1 = n2 = n3 = -1 (that is, run to fixpoint) or that *SatElite*-like preprocessing is not to be run at all, respectively.

Having introduced the general options of clasp, let us note that the options below --supp-models in the above list are quite low-level and more or less an issue of fine-tuning. More important is the fact that virtually all optimization functionalities are only provided by clasp if the maximum number of answer sets to be computed is set to 0 (standing for all answer sets), as it is likely to stop search too early otherwise. The same applies to computing either brave or cautious consequences (via one of the flags --brave and --cautious).

#### 5.4.2 Search Options

The options listed below can be used to configure the main search strategies of clasp.

#### --lookahead=atom|body|hybrid|no

Apply failed-literal detection [17] to atoms (with argument atom), to rule bodies (with argument body), or to atoms and rule bodies like in nomore++ [1] (with argument hybrid). Failed-literal detection is switched off via argument no.

#### --initial-lookahead=n

Apply failed-literal detection in preprocessing, and for the first *n* decisions during search.

#### --heuristic=Berkmin|Vmtf|Vsids|Unit|None

Use *BerkMin*-like decision heuristic [31] (with argument Berkmin), *Siege*-like decision heuristic [48] (with argument Vmtf), *Chaff*-like decision heuristic [41] (with argument Vsids), *Smodels*-like decision heuristic [51] (with argument Unit), or (arbitrary) static variable ordering (with argument None).

#### --rand-freq=p

Perform random (rather than heuristic) decisions with probability  $0 \le p \le 1$ .

#### --rand-prob=yes|no|n1,n2

Run Satzoo-like random probing [9], initially performing n1 passes of up to n2 conflicts making random decisions. Arguments yes and no mean n1 = 50, n2 = 20 or that random probing is not to be run at all, respectively.

#### --rand-watches=yes|no

Initially choose watched literals randomly (with argument yes) or systematically (with argument no).

#### 5.4.3 Lookback Options

The following options have an effect only if lookback techniques are turned on, that is, option --no-lookback is not used.

#### --no-lookback

Disable all lookback techniques. This option is included mainly for comparison purposes, and its use is not generally recommended.

#### --restarts, -r *n1*[, *n2*, *n3*] |no

Choose and parameterize a restart policy. If a single argument n1 is provided, clasp restarts search from scratch after a number of conflicts determined by a universal sequence [38], where n1 constitutes the base unit. If two arguments n1, n2 are specified, clasp runs a geometric sequence [11], restarting every  $n1 * n2^i$  conflicts, where *i* is the number of restarts performed so far. Given three arguments n1, n2, n3, clasp repeats geometric restart sequence  $n1*n2^i$  when it reaches an outer limit  $n3 * n2^j$  [5], where *j* counts how often the outer limit has been hit so far. Finally, restarts are disabled via argument no.

#### --local-restarts

Count conflicts locally [49] (rather than globally) for deciding when to restart.

#### --bounded-restarts

Perform bounded restarts in answer set enumeration [21].

#### --reset-restarts

Reset restart strategy whenever an answer set is found.

#### --save-progress[=n]

Use cached (rather than heuristic) decisions [45] if available. Cache decisions on backjumps > n.

#### --shuffle,-s n1,n2

Shuffle internal data structures after n1 restarts (n1 = 0 standing for no shuffling) and then reshuffle every n2 restarts (n2 = 0 standing for no reshuffling).

#### --deletion,-d n1[,n2,n3]

Limit the number of learnt constraints to  $\min\{(c/n1) * n2^i, c * n3\}$ , where c is the initial number of constraints and i is the number of restarts performed so far.

#### --reduce-on-restart

Delete a portion of learnt constraints after every restart.

#### --estimate

Base the initial limit of learnt constraints on an estimate of the problem's complexity.

#### --strengthen=bin|tern|all|no

Check binary (with argument bin), binary and ternary (with argument tern), or all (with argument all) antecedents for self-subsumption [10] in order to strengthen a constraint to learn. Strengthening is disabled via argument no.

#### --recursive-str

Recursively apply strengthening, as proposed in [52].

#### --loops=common|distinct|shared|no

Learn loop nogood [22] per atom in an unfounded set [55] (with argument common), shrink unfounded set before learning another loop nogood (with argument distinct), learn loop formula [37] for atoms in an unfounded set (with argument shared), or do not record unfounded sets at all (with argument no).

#### --contraction=n

Temporarily truncate learnt constraints over more than *n* variables [48].

Let us note that switching the above search and lookback options can have dramatic effects (both positively and negatively) on the search performance of clasp. If performance bottlenecks are observed, it is worthwhile to give Vmtf and Vsids for --heuristic a try, in particular, when the program under consideration is huge but scarcely yields conflicts during search. Furthermore, we suggest trying the universal restart sequence [38] with different base units n1 or even disabling restarts (both via (long) option --restarts or its short form -r) in case that performance needs to be improved. For a brief overview on fine-tuning see [24]. Finally, let us consider the default command line of clasp:

```
clasp 1 --verbose=1 --seed=1 --trans-ext=no --eq=5
    --sat-prepro=no --lookahead=no
    --heuristic=Berkmin --deletion=3.0,1.1,3.0
    --rand-freq=0.0 --rand-watches=yes --rand-prob=no
    --restarts=100,1.5 --shuffle=0,0 --strengthen=all
    --loops=common --contraction=250
```

Considering only the most significant defaults, numeric argument 1 instructs clasp to terminate immediately after finding an answer set, while --restarts=100, 1.5 lets clasp apply a geometric restart policy.

## 6 Errors and Warnings

This section explains the most frequent errors and warnings related to inappropriate inputs or command line options that, if they occur, lead to messages sent to the standard error stream. The difference between errors and warnings is that the former involve immediate termination, while the latter are hints pointing at possibly corrupt input that can still be processed further. In the below description of errors (Section 6.1) and warnings (Section 6.2), we refrain from attributing them to a particular one among the tools gringo, clasp, clingo, and iclingo, in view of the fact that they share a number of functionalities.

## 6.1 Errors

We start our description with errors that may be encountered during grounding, where the following one indicates a syntax error in the input:

ERROR: parsing failed: *File:Line:Column:* unexpected token: *Token* 

To correct this error, please investigate the line *Line* and check whether something looks strange there (like a missing period, an unmatched parenthesis, etc.).

The next error occurs if an input program is not safe:

```
ERROR: unsafe variables in:
File:Line:Column: Rule
File:Line:Column: Var
```

Along with the error message, the *Rule* and the name *Var* of at least one variable causing the problem are reported. The first action to take usually consists of checking whether variable *Var* is actually in the scope of any atom (in the positive body of *rule*) that can bind it.<sup>18</sup> If *Var* is a local variable belonging to an atom A on the left-hand side of a condition (cf. Section 3.1.8) or to an aggregate (cf. Section 3.1.10), an atom over some domain predicate might be included in a condition to bind *Var*. In particular, if A itself is over a domain predicate, the problem is often easily fixed by writing "A: A."

The following error is related to conditions (cf. Section 3.1.8):

```
ERROR: unstratified predicate in:

File:Line:Column: Rule

File:Line:Column: Predicate/Arity
```

The problem is that an atom *Predicate(...)* such that its predicate *Predicate/Arity* is not a domain predicate (cf. Section 3.1.8) is used on the right-hand side of a condition within *Rule*. The error is corrected by either removing the atom or by replacing it with another atom over a domain predicate.

The next errors may occur within an arithmetic evaluation (cf. Section 3.1.4):

```
ERROR: cannot convert Term to integer in:
File:Line:Column: Literal
```

It means that either a (symbolic) constant or a compound term (over an uninterpreted function with non-zero arity) has occurred in the scope of some built-in arithmetic function.

The following error message is issued by (embedded) clasp:

ERROR: Read Error: Line 2, Compute Statement expected!

This error means that the input does not comply with <code>lparse's</code> numerical format [53]. It is not unlikely that the input can be processed by <code>gringo</code>, <code>clingo</code>, or <code>iclingo</code>.

The next error indicates that input in lparse's numerical format [53] is corrupt:

ERROR: Read Error: Line Line, Atom out of bounds

There is no way to resolve this problem. If the input has been generated by gringo, clingo, or iclingo, please report the problem to the authors of this guide.

The following error message is issued by (embedded) clasp:

ERROR: Read Error: Line Line, Unsupported rule type!

It means that some rule type in lparse's numerical format [53] is not supported. Most likely, the program under consideration contains rules with disjunction in the head.

A similar error may occur with clingo or iclingo:

ERROR: Error: clasp cannot handle disjunctive rules, \ use option --shift!

<sup>&</sup>lt;sup>18</sup>Recall from Section 3.1.4 and 3.1.5 that a variable in the scope of a built-in arithmetic function may not be bound by a corresponding atom and that built-in comparison predicates do not bind any variable.

The program under consideration contains rules with disjunction in the head, which are currently not supported by clasp, but by claspD [8]. The integration of clasp and claspD is a subject to future work (cf. Section 7). Furthermore, if your program is head-cycle free, you might want to try gringo's --shift option (see 5.1).

All of the tools gringo, clasp, clingo, and iclingo try to expand incomplete (long) options to recognized ones. Parsing command line options may nonetheless fail due to the following three reasons:

```
ERROR: unknown option: Option
ERROR: ambiguous option: 'Option' could be:
    Option1
    Option2
    ...
ERROR: 'Arg': invalid value for Option 'Option'
```

The first error means that a provided option *Option* could not be expanded to one that is recognized, while the second error expresses that the result of expanding *Option* is ambiguous. Finally, the third error occurs if a provided argument *Arg* is invalid for option *Option*. In either case, option --help can be used to see the recognized options and their arguments.

## 6.2 Warnings

The following warnings may be raised by gringo, clingo, or iclingo:

```
% warning: p/i is never defined
```

This warning, states that a predicate p/i has occurred in some rule body, but not in the head of any rule, might point at a mistyped predicate.

# 7 Future Work

We conclude this guide with a brief outlook on the future development of gringo, clasp, clingo, and iclingo. An important goal of future releases will be improving usability by adding functionalities that make some errors and warnings obsolete or, otherwise, by providing helpful context information along with the remaining ones. In particular, we consider adding support for yet missing traditional features in incremental computations of iclingo and also the integration of clasp and claspD [8], which would enable clasp, clingo, and iclingo to deal with disjunctive programs (cf. [12]) or, more generally, logic programs such that standard reasoning tasks are complete for the second level of the polynomial hierarchy (cf. [44]). Moreover, we are investigating less demanding restrictedness notions that can broaden the class of acceptable input programs. For the representation of ground programs, ASPils format [18] has been suggested to overcome limitations of lparse's output format [53], and we work on ASPils support in clasp. In the long term, limitations inherent to present ASP systems, such as space explosion sometimes faced when representing multi-valued variables in a propositional formalism, might be extinguished by systems combining ASP with neighboring paradigms like, e.g., Constraint Programming [47]. Prototypical approaches in such directions already exist today [40, 43].

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## A Differences to the Language of lparse

We below provide a (most likely incomplete) list of differences between the input languages of gringo [26] and lparse [53]. First of all, it is important to note that the language of gringo significantly extends the one of lparse. For instance, #min, #max, #even, #odd, and #avg aggregates (cf. Section 3.1.10) are not supported by lparse. Furthermore, gringo provides full support for variables within compound terms (over uninterpreted functions with non-zero arity). That is, an atom like p(f(X)) in the positive body of a rule can potentially be used to bind X (cf. Section 3.1.1), while lparse would treat f(X) like an arithmetic function (cf. Section 3.1.4) whose variables cannot be bound. Finally, gringo deals with safe programs, while lparse requires programs to be  $\omega$ -restricted [54]. As the latter are more restrictive, programs for lparse tend to be more verbose than the ones for gringo. Thus, we do not suggest writing programs in the input language of gringo for compatibility to lparse.

However, a bulk of existing encodings are written for lparse (see, e.g., [3]), and gringo (likewise, clingo and iclingo) should actually be able to deal with most of them. If this is not case, one of the following might be the reason:

- The input contains primed atoms like p', which are (currently) not supported by gringo.
- Symbolic names are used for built-in constructs, e.g., plus or eq for built-in arithmetic function + or predicate ==, respectively. Such names are not associated to built-in constructs by gringo, as they may accidentally clash with users' names otherwise.
- The input contains (or is instantiated to) a #count aggregate (or a cardinality constraints, respectively) containing duplicates of literals. Such duplicates are not removed by lparse, e.g., it treats 2{p(c),p(c)} as a synonym for 2[p(c)=1,p(c)=1]. In contrast, gringo associates curly brackets to a set (and square brackets to a multiset), as described in Section 3.1.10, so that 2{p(c),p(c)} is the same as 2{p(c)}, where the latter can clearly not hold.
- Pooling is expanded differently, e.g., lparse interprets p(X, Y; X, Z) as a shorthand for p(X, Y), p(X, Z), while gringo expands it to p(X, Y, Z), p(X, X, Z), as explained in Section 3.1.9.

As indicated above, the provided list is probably incomplete. If you would like some difference(s) to be added, please contact the authors of this guide.

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